The Generalized Weyl Poisson algebras and their Poisson simplicity criterion

Volodymyr Bavula

A new large class of Poisson algebras, the class of generalized Weyl Poisson algebras, is introduced. It can be seen as Poisson algebra analogue of generalized Weyl algebras. A Poisson simplicity criterion is given for generalized Weyl Poisson algebras and an explicit description of the Poisson centre is obtained. Many examples are considered.

References

CONTACT INFORMATION

Volodymyr Bavula
School of Mathematics and Statistics, University of Sheffield, Sheffield, UK
Email address: v.bavula@sheffield.ac.uk

Key words and phrases. The Generalized Weyl Poisson algebras, a Poisson simplicity criterion, a Poisson ideal, the classical polynomial Poisson algebra

The specialized characters of the representation of the Lie algebra $sl_3$ in terms of $q$- and $(q,p)$-numbers

Leonid Bedratyuk, Ivan Kachuryk

Let $\Gamma_\lambda$ be the standard irreducible complex representation of $sl_3$ with the highest weight $\lambda = (\lambda_1, \lambda_2) \in \mathbb{Z}^2$, $\dim \Gamma_\lambda = (\lambda_1 + 1)(\lambda_2 + 1)(\lambda_1 + \lambda_2)/2$.

Denote by $\Lambda$ the weight lattice of all finite dimensional representation of $sl_3$, and let $\mathbb{Z}(\Lambda)$ be their group ring. The ring $\mathbb{Z}(\Lambda)$ is free $\mathbb{Z}$-module with the basis elements $e(\lambda)$, $\lambda = (\lambda_1, \lambda_2) \in \Lambda$, $e(\lambda)e(\mu) = e(\lambda + \mu)$, $e(0) = 1$. Let $\Lambda_\lambda$ be the set of all weights of the representation $\Gamma_\lambda$. Then the formal character $\text{Char}(\Gamma_\lambda)$ is defined as formal sum $\sum_{\mu \in \Lambda_\lambda} \nu_\lambda(\mu)e(\mu) \in \mathbb{Z}(\Lambda)$, here $n_\lambda(\mu)$ is the multiplicities of the weight $\mu$ in the representation $\Gamma_\lambda$. By replacing $e(m,n) := q^m p^n$ we obtain the specialized expression for the character of $\text{Char}(\Gamma_{(n,m)}) \equiv [n,m]_{qp}$.

We establish several relations between the specialized characters $[n,m]_{qp}$ and the quantum $(q,p)$-numbers

$$[r]_{qp} = \frac{q^r - p^{-r}}{q - p^{-1}},$$

and in some cases between different types of $q$-numbers.
We obtain the following expression
\[ [n, 0]_{q, p} - (pq)^{-1} [n - 1, 0]_{q, p} = [n + 1]_{q, p}, \quad [-1, 0]_{q, p} = 0, [0, 0]_{q, p} = 1. \]
As a consequence, we obtain that
\[ [n, 0]_{q, p} = \sum_{k=1}^{n+1} (pq^{-1})^{n-k+1} [k]_{q, p}. \]
By \( p = q \) we get \([n, 0]_{q, q} - [n - 1, 0]_{q, q} = [n + 1]_{q, q} \), where \([n]_q \equiv [n]_{q, q} \) is the \( q \)-number and
\[ [n, 0]_{q, q} = \sum_{k=1}^{n+1} [k]_q. \]
Further calculations lead to the formulas
\[ [n - 1, 0]_{q, q} = \frac{[n + 1]_{q, q} - (pq^{-1})^2 [q, q]_p - (pq^{-1})^n [1]_{q, q}}{[2]_{q, q} - (pq^{-1})^2 - (pq^{-1})}, \]
\[ [0, n - 1]_{q, q} = \frac{(pq^{-1})^{n-1} [n + 1]_{q, q} - (pq^{-1})^n [n]_{q, q} - (pq^{-1})^{n+1}}{[2]_{q, q} - (pq^{-1})^2 - (pq^{-1})}. \]
In particular we find
\[ [n - 1, 0]_{q, q} = [0, n - 1]_{q, q} = \frac{[n]_{q, q} [n + 1]_{q, q}^2}{[2]_{q, q}^2}, \]
\[ \lim_{q^{-1} \to 1} [n - 1, 0]_{q, q} = \frac{n(n + 1)}{2}. \]
It turns out that in the general case the characters \([n, m]_{q, p} \) can also be represented through \((q, p)\)-numbers \([n]_{q, p} \), and in partial cases, through known in theoretical and mathematical physics of different types of \( q \)-numbers, which are considered as exponential deformations of the usual \( c \)-number \( r \). To show this, we use the theorem
\[ [n, m]_{q, p} = [n, 0]_{q, p} [0, m]_{q, p} - [n - 1, 0]_{q, p} [0, m - 1]_{q, p}, \]
and obtain the following result
\[ [n, m]_{q, p} = [m, n]_{p, q} = (pq^{-1})^m [n + m + 2]_{q, p} - (pq^{-1})^{2(m+1)} [n + 1]_{q, p} - (pq^{-1})^{n+1} [m + 1]_{q, p}. \]
By \( m = n \) we get
\[ [n, n]_{q, p} = [n + 1]_{q, p} [n + 1]_{q, p} - [n + 1]_{q, p} [n + 1]_{q, p}^{-1}. \]
If \( p \to q^{-1} \) then the specialized characters can be expressed in terms of \( q \)-deformed numbers of the form \( q_{r}^{-1} \cdot r (= \lim_{p \to q^{-1}} [r]_{q, p}) \):
\[ [n, m]_{q, q}^{-1} = \lim_{p \to q^{-1}} [n, m]_{q, q} = \frac{q^{-n+\frac{m+3}{2}} (n + 1)[m + 1]_{q, q}^2 - q^{-n+\frac{m+3}{2}} (m + 1)[n + 1]_{q, q}^2}{q^{-3/2} - q^{-3/2}}. \]
Also we have
\[ [n - 1, 0]_{q, q}^{-1} = q^{-2(n-1)} \frac{1 - q^{3n} (n(1 - q^3) + 1)}{(1 - q^3)^2} = [0, n - 1]_{q, q}^{-1} \bigg|_{q = q^{-1}}, \]
\[ [n, n]_{q, q}^{-1} = (n + 1)[n + 1]_{q, q}^2 \]
For the case \( p = q \) the specialized characters also can be expressed in terms of \( q \)-deformed numbers \([r]_q = [r]_{q, q} = (q^a - q^{-n})/\sqrt{q - q^{-1}} \):
\[ [n, m]_{q, q} = \frac{[n + 1]_{q, q}^2 [m + 1]_{q, q}^2 [n + m + 2]_{q, q}^2}{[2]_{q, q}^2}, \]
\[ [n, n]_{q, q} = \frac{[n + 1]_{q, q}^2 [2(n + 1)]_{q, q}^2}{[2]_{q, q}^2}. \]
For the limit $q \to q^{-1}$ we get

$$\{n, m\} \equiv \lim_{q \to 1} [n, m]_{q, q} = \lim_{q \to 1} [n, m]_{q, q^{-1}} = \frac{1}{2} (n + 1)(m + 1)(n + m + 2) = \dim \Gamma_{n,m},$$

$$\{n - 1, n - 1\} = n^3 = \dim \Gamma_{n-1,n-1},$$

$$\{n - 1, 0\} = \{0, n - 1\} = \frac{n(n+1)}{2} = \dim \Gamma_{n-1,0}$$

For $p \to 1$ the $(q, p)$-numbers $[r]_{q,p}$ turn into the Jackson $q$-numbers $[r]_q \equiv (1 - q^n)/(1 - q).$

We prove that

$$[n, m]_{q, 1} = q^{-(n+m)} \frac{[n + m + 2]_q [n + 1]_q [m + 1]_q}{[2]_q},$$

$$[n, m]_{q, 1} = q^{-2n} \frac{[n + 1]_q [2(n + 1)]_q}{[2]_q},$$

$$[n - 1, 0]_{q, 1} = \frac{q^{-n} [n]_q [n + 1]_q}{[2]_q}.$$

References


Contact information

Leonid Bedratyuk
Faculty of Programming, Computer and Telecommunication Systems, Khmelnytskyi National University, Khmelnytskyi, Ukraine
Email address: leonid.uk@gmail.com

Ivan Kachuryk
Department of Computer Science, Glukhiv National Pedagogical University, Glukhiv, Ukraine

Some properties of generalized hypergeometric Appell polynomials

Leonid Bedratyuk, Natalia Luno

In [1], P. Appell presented the sequence of polynomials $\{A_n(x)\}, n = 0, 1, 2, \ldots$ which satisfies the following relation

$$A'_n(x) = nA_{n-1}(x),$$

and possesses the exponential generating function

$$A(t)e^x = \sum_{n=0}^{\infty} A_n(x) \frac{t^n}{n!},$$

where $A(t)$ is a formal power series

$$A(t) = a_0 + a_1 t + a_2 \frac{t^2}{2!} + \cdots + a_n \frac{t^n}{n!} + \cdots, a_0 \neq 0.$$

The Appell type polynomials appear at the different areas of mathematics, namely, at special functions, general algebra, combinatorics and number theory. Recently, the Appell type