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The Generalized Weyl Poisson algebras and their Poisson simplicity criterion

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A new large class of Poisson algebras, the class of generalized Weyl Poisson algebras, is introduced. It can be seen as Poisson algebra analogue of generalized Weyl algebras. A Poisson simplicity criterion is given for generalized Weyl Poisson algebras and an explicit description of the Poisson centre is obtained. Many examples are considered.

References

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The specialized characters of the representation of the Lie algebra sl_3 in terms of q- and (q,p)-numbers

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Let Γ_λ be the standard irreducible complex representation of $\mathfrak{sl}_3$ with the highest weight $\lambda = (\lambda_1, \lambda_2) \in \mathbb{Z}^2$, $\dim \Gamma_\lambda = (\lambda_1+1)(\lambda_2+1)(\lambda_1+\lambda_2)/2$.

Denote by Λ the weight lattice of all finite dimensional representation of $\mathfrak{sl}_3$, and let $\mathbb{Z}(\Lambda)$ be their group ring. The ring $\mathbb{Z}(\Lambda)$ is free $\mathbb{Z}$-module with the basis elements $e(\lambda)$, $\lambda = (\lambda_1, \lambda_2) \in \Lambda$, $e(\lambda)e(\mu) = e(\lambda+\mu)$, $e(0) = 1$. Let Λ_λ be the set of all weights of the representation Γ_λ. Then the formal character $\mathrm{Char}(\Gamma_\lambda)$ is defined as formal sum $\sum_{\mu \in \Lambda_\lambda} n_\lambda(\mu) e(\mu) \in \mathbb{Z}(\Lambda)$, here $n_\lambda(\mu)$ is the multiplicities of the weight μ in the representation Γ_λ. By replacing $e(m,n) := q^n p^m$ we obtain the specialized expression for the character of $\mathrm{Char}(\Gamma_{(n,m)}) \equiv [n,m]_{q,p}$.

We establish several relations between the specialized characters $[n,m]_{qp}$ and the quantum (q,p)-numbers

$$[r]_{q,p} = \frac{q^r - p^{-r}}{q - p^{-1}},$$

and in some cases between different types of q-numbers.

We obtain the following expression

$$[n,0]_{q,p}-(pq)^{-1}[n-1,0]_{q,p}=[n+1]_{q,p},\ \ [-1,0]_{q,p}=0,[0,0]_{q,p}=1.$$

As a consequence, we obtain that

$$[n,0]_{q,p}=\sum_{k=1}^{n+1}(pq^{-1})^{n-k+1}[k]_{q,p}.$$

By $p=q$ we get $[n,0]_{q,q}-[n-1,0]_{q,q}=[n+1]_{q,q}$, where $[n]_q\equiv[n]_{q,q}$ is the q-number and

$$[n,0]_{q,q}=\sum_{k=1}^{n+1}[k]_q.$$

Further calculations lead to the formulas

$$[n-1,0]_{q,p}=\frac{[n+1]_{q,p}-(qp^{-1})^2[n]_{q,p}-(pq^{-1})^n[1]_{q,p}}{[2]_{q,p}-(qp^{-1})^2-(pq^{-1})},$$

$$[0,n-1]_{q,p}=\frac{(pq^{-1})^{n-1}[n+1]_{q,p}-(pq^{-1})^n[n]_{q,p}-(qp^{-1})^{n+1}}{[2]_{q,p}-(qp^{-1})^2-(pq^{-1})}.$$

In particular we find

$$[n-1,0]_{q,q}=[0,n-1]_{q,q}=\frac{[n]_{q^{1/2}}[n+1]_{q^{1/2}}}{[2]_{q^{1/2}}},$$

$$\lim_{q\to1}[n-1,0]_{q,q}=\frac{n(n+1)}{2}.$$

It turns out that in the general case the characters $[n,m]_{qp}$ can also be represented through (q,p)-numbers $[n]_{qp}$, and in partial cases, through known in theoretical and mathematical physics of different types of q -numbers, which are considered as exponential deformations of the usual c-number r. To show this, we use the theorem

$$[n,m]_{q,p}=[n,0]_{q,p}[0,m]_{q,p}-[n-1,0]_{q,p}[0,m-1]_{q,p},$$

and obtain the following result

$$[n,m]_{q,p}=[m,n]_{p,q}=(pq^{-1})^m\frac{[n+m+2]_{q,p}-(qp^{-1})^{2(m+1)}[n+1]_{q,p}-(pq^{-1})^{n+1}[m+1]_{q,p}}{[2]_{q,p}-(qp^{-1})^2-(pq^{-1})}.$$

By $m=n$ we get

$$[n,n]_{q,p}=[n+1]_{q,p}[n+1]_{qp^{-\frac{1}{2}}}[n+1]_{pq^{-\frac{1}{2}}}$$

If $p\to q^{-1}$ then the specialized characters can be expressed in terms of q-deformed numbers of the form $q^{r-1}r(=\lim_{p\to q^{-1}}[r]_{qp}$:

$$[n,m]_{qq^{-1}}=\lim_{p\to q^{-1}}[n,m]_{pq}=\frac{q^{n+\frac{m+3}{2}}(n+1)[m+1]_{q^{3/2}}-q^{-n-\frac{m+3}{2}}(m+1)[n+1]_{q^{3/2}}}{q^{3/2}-q^{-3/2}}.$$

Also we have

$$[n-1,0]_{q,q^{-1}}=q^{-2(n-1)}\frac{1-q^{3n}(n(1-q^3)+1)}{(1-q^3)^2}=[0,n-1]_{q,q^{-1}}\Big|_{q\leftrightarrow q^{-1}}$$

$$[n,n]_{q,q^{-1}}=(n+1)[n+1]^2_{q^{3/2}}$$

For the case $p=q$ the specialized characters also can be expressed in terms of q-deformed numbers $[r]_q=[r]_{q,q}=(q^n-q^{-n})/(q-q^{-1})$:

$$[n,m]_{q,q}=\frac{[n+1]_{q^{1/2}}[m+1]_{q^{1/2}}[n+m+2]_{q^{1/2}}}{[2]_{q^{1/2}}},$$

$$[n,n]_{q,q}=\frac{[n+1]^2_{q^{1/2}}[2(n+1)]_{q^{1/2}}}{[2]_{q^{1/2}}}.$$

For the limit $q \to q^{-1}$ we get

$$\{n,m\} \equiv \lim_{q\to 1}[n,m]_{q,q} = \lim_{q\to 1}[n,m]_{q,q^{-1}} = \frac{1}{2}(n+1)(m+1)(n+m+2) = \dim \Gamma_{n,m},$$
$$\{n-1,n-1\} = n^3 = \dim \Gamma_{n-1,n-1},$$
$$\{n-1,0\} = \{0,n-1\} = \frac{n(n+1)}{2} = \dim \Gamma_{n-1,0}$$

For $p \to 1$ the (q,p)-numbers $[r]_{q,p}$ turn into the Jackson q-numbers $[r)_q \equiv (1-q^n)/(1-q)$. We prove that

$$[n,m]_{q,1} = q^{-(n+m)}\frac{[n+m+2)_q[n+1)_q[m+1)_q}{[2]_q},$$
$$[n,m]_{q,1} = q^{-2n}\frac{[n+1)_q^2[2(n+1))_q}{[2]_q},$$
$$[n-1,0]_{q,1} = \frac{q^{-n}[n)_q[n+1)_q}{[2]_q}.$$

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Some properties of generelized hypergeometric Appell polynomials

Leonid Bedratyuk, Nataliia Luno

In [**1**], P.Appell presented the sequence of polynomials $\{A_n(x)\}, n = 0,1,2,\dots$ which satisfies the following relation

$$A_n'(x) = nA_{n-1}(x),$$

and possesses the exponential generating function

$$A(t)e^{xt} = \sum_{n=0}^{\infty} A_n(x)\frac{t^n}{n!},$$

where $A(t)$ is a formal power series

$$A(t) = a_0 + a_1 t + a_2\frac{t^2}{2!} + \cdots + a_n\frac{t^n}{n!} + \cdots \quad , a_0 \neq 0.$$

The Appell type polynomials appear at the different areas of mathematics, namely, at special functions, general algebra, combinatorics and number theory. Resently, the Appell type