Corollary 1. Transforming matrices $U$ and $V$ from (2) have the following upper unitriangular form

$$U = \begin{bmatrix} I & -Y \\ 0 & I \end{bmatrix}, \quad V = \begin{bmatrix} I & -X \\ 0 & I \end{bmatrix},$$

where matrices $X$ and $Y$ have the same triangular form as matrices $A, B$ and $C$ if and only if $(a_{ii}, b_{ii})|c_{ii}$ for all $i = 1, 2, \ldots, n$.

References


Contact Information

Nataliia Dzhaliuk
Department of Algebra,Pidstryhach Institute for Applied Problems of Mechanics and Mathematics of the NAS of Ukraine, L’viv, Ukraine
Email address: nataliya.dzhalyuk@gmail.com

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Some notes on orthogonality

Iryna Fryz

A tuple of $n$-ary operations $f_1, \ldots, f_k$ ($n \geq 2, k \leq n$) defined on a set $Q$ ($m := |Q|$) is called orthogonal [1], if for arbitrary $b_1, \ldots, b_k \in Q$ the system $\{f_i(x_1, \ldots, x_n) = b_i\}_{i=1}^k$ has exactly $m^{n-k}$ solutions.

Let $f$ be an $n$-ary operation on $Q$ and

$$\delta := \{i_1, \ldots, i_k\} \subset \overline{1, n} := \{1, \ldots, n\}, \quad \{j_1, \ldots, j_{n-k}\} := \overline{1, n} \setminus \delta, \quad \bar{a} := (a_{j_1}, \ldots, a_{j_{n-k}}).$$

An operation $f_{(\bar{a}, \delta)}$ which is defined by

$$f_{(\bar{a}, \delta)}(x_{i_1}, \ldots, x_{i_k}) := f(y_1, \ldots, y_n),$$

where $y_i := \begin{cases} x_i, & \text{if } i \in \delta, \\ a_i, & \text{if } i \notin \delta. \end{cases}$ is called an $(\bar{a}, \delta)$-retract or a $\delta$-retract of $f$. Operations $f_{1:(\bar{a}_1, \delta)}, \ldots, f_{k:(\bar{a}_k, \delta)}$ are called similar $\delta$-retracts of $n$-ary operations $f_1, \ldots, f_k$, if $\bar{a}_1 = \cdots = \bar{a}_k$. A $k$-tuple of $n$-ary operations is called $\delta$-retractly orthogonal [4], if all tuples of similar $\delta$-retracts of these operations are orthogonal.

The notion of perpendicularity of the maximal type from [3] can be defined using the definition of retract orthogonality: $n$-ary operations $g$ and $h$ are called perpendicular of the type $(\iota, \iota; m)$, if they are $\delta$-retractly orthogonal for all $\delta$ such that $|\delta| = 2$ and $m \in \delta$. The results from [5] imply the following statement.

Proposition 1. If $n$-ary operations $g$ and $h$ are $\delta$-perpendicular of the type $(\iota, \iota; m)$, $m \in \overline{1, n}$, then they are $\delta$-retractly orthogonal for all $\delta \subset \overline{1, n}$, where $|\delta| > 1$ and $m \in \delta$.

The relationships between retract orthogonality and strong orthogonality was described by G.B. Belyavskaya and G.L. Mullen [2] and the relationships between retract orthogonality and orthogonality was studied in [5].
**Proposition 2.** Let $g$ and $h$ be $n$-ary quasigroups. The following statements are equivalent:

1. $g$ and $h$ are strongly orthogonal;
2. $g$ and $h$ are perpendicular of the type $(i, i; m)$ for all $m \in \overline{1,n}$;
3. $g$ and $h$ are $\delta$-retractly orthogonal for all $\delta \subset \overline{1,n}$;
4. for an arbitrary $m \in \overline{1,n}$ operation $g \oplus h$ is invertible, where
   \[(g \oplus h)(x_1, \ldots, x_n) := g(x_1, \ldots, x_{m-1}, h(x_1, \ldots, x_n), x_{m+1}, \ldots, x_n).\]

**References**


**Contact information**

Iryna Fryz
Vasyl’ Stus Donetsk National University, Vinnytsia, Ukraine
*Email address*: iryna.fryz@ukr.net

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**Diagonal reduction of matrices over commutative semihereditary Bezout rings**

**Andrii Gatalevych**

All rings considered will be commutative and have identity. Recently there has been some interest in the polynomial ring $R[x]$, where $R$ is a von Neumann regular ring. Such a ring is a Bezout ring, semihereditary ring, and so Hermite ring. Thus, it is natural to ask whether or not $R[x]$ is an elementary divisor ring. This question is answered affirmative in [3]. It is an open problem whether or not every Bezout domain is an elementary divisor ring and more generally: whether or not every semihereditary Bezout ring is an elementary divisor ring.

We obtain a complete characterization of semihereditary elementary divisor ring through its homomorphic images.

Mc Adam S. and Swan R. G. studied comaximal factorization in commutative rings [2]. Following them, we give the following definitions.

**Definition 1.** A nonzero element $a$ of a ring $R$ is called inpseudo-irreducible if for any representation $a = b \cdot c$ we have $bR + cR = R$.

**Definition 2.** An element $a$ of a ring $R$ is called pseudo-irreducible if for any representation $a = b \cdot c$, where $b, c \not\in U(R)$, we have $bR + cR \neq R$.

Other definitions can be found in the articles [1, 4].

**Theorem 1.** Let $R$ be a Bezout ring of stable range 2. A regular element $a \in R$ is inpseudo-irreducible if $R/aR$ is a von Neumann regular ring.