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On eigenvalues of random partial wreath product

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Partial wreath $n$-th power of symmetric inverse semigroup $\mathcal{I}_d$ is a semigroup defined recursively by

$$\mathcal{P}_n = (\mathcal{P}_{n-1}) \wr \mathcal{I}_d = \{(f, a) | a \in \mathcal{I}_d, f : \text{dom}(a) \to \mathcal{P}_{n-1}\}, n \geq 2,$$

with composition

$$(f, a) \cdot (g, b) = (fg^a, ab),$$

and $\mathcal{P}_1 = \mathcal{I}_d$.

To a randomly chosen element $x \in \mathcal{P}_n$, we assign the matrix

$$A_x = \left(1_{\{x(v^n_i) = v^n_j\}}\right)_{i,j=1}^{d^n}.$$

In other words, $(i, j)$-th entry of $A_x$ is equal to 1 if transformation $x$ maps $i$ to $j$, and 0 otherwise.

Let $\chi_x(\lambda)$ be the characteristic polynomial of $A_x$ and $\lambda_1, \ldots, \lambda_{d^n}$ be its roots respecting multiplicity.

Denote by

$$\Xi_n = \frac{1}{d^n} \sum_{k=1}^{d^n} \delta_{\lambda_k}$$

a uniform distribution on eigenvalues of $A_x$.

Let $x \in \mathcal{P}_n$ let $\eta_n(x) = \Xi_n(0)$ be a fraction of zero eigenvalues $A_x$ and $\xi_n(x) = 1 - \eta_n(x)$ be a fraction of nonzero eigenvalues. Then we have

$$\eta_n(x) \xrightarrow{P} 1, n \to \infty,$$

or, equivalently,

$$\xi_n(x) \xrightarrow{P} 0, n \to \infty.$$

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Classification of quasigroup functional equations and identities of minimal length

HALYNA KRAINICHUK

A groupoid \((Q; \cdot)\) is called a *quasigroup*, if for all \(a, b \in Q\) every of the equations \(x \cdot a = b\) and \(a \cdot y = b\) has a unique solution. A \(\sigma\)-parastrophe \((Q; \sigma \cdot)\) of \((Q; \cdot)\) is defined by
\[
x_1 \sigma \cdot x_2 \sigma = x_3 \sigma \iff x_1 \cdot x_2 = x_3, \quad \sigma \in S_3.
\]

A \(\sigma\)-parastrophe of a class of quasigroups \(\mathfrak{A}\) is called a class \(\sigma \mathfrak{A}\), which consists of all \(\sigma\)-parastrophes of quasigroups from \(\mathfrak{A}\) [4].

Two identities are called:

— equivalent, if they determine the same variety;
— parastrophically equivalent, if they determine parastrophic varieties.

Evidently that every equinelement identity are parastrophically equivalent, but the inverse is not valid.

A parastrophic symmetry group of a variety \(\mathfrak{A}\) is \(\text{Ps}(\mathfrak{A}) := \{\sigma \mid \sigma \mathfrak{A} = \mathfrak{A}\}\) and it is subgroup of the group \(S_3\). A variety is called:

— totally-symmetric, if \(\text{Ps}(\mathfrak{A}) = S_3\);
— semisymmetric, if \(\text{Ps}(\mathfrak{A}) = A_3\);
— one-sided-symmetric, if \(|\text{Ps}(\mathfrak{A})| = 2|\);
— asymmetric, if \(|\text{Ps}(\mathfrak{A})| = 1|\).

A truss of varieties is called the set of all pairwise parastrophic varieties. A truss of varieties is uniquely defined by an identity which describes one of varieties from the given truss. A truss will be called: *totally-symmetric*, if it has 1 variety; *semisymmetric*, if it has two varieties; *one-sided-symmetric*, if it has three varieties; *asymmetric*, if it has six varieties.

A length of an identity is defined as the number of all functional symbols (not necessary different) appearing in it. Any quasigroup identity of length 1 is equivalent to the identity of idempotency.

**Theorem 1.** An arbitrary quasigroup identity of length 2 is equivalent to exactly one of the following 14 identities and is parastrophically-equivalent to exactly one of the 6 identities having different numbers:

\[
\begin{align*}
1) & \quad x = x, & 2) & \quad xy \cdot x = y, \\
3) & \quad xy = yx, & 4) & \quad x^2 = y^2, & 5) & \quad x^2 \cdot y = y, & 6) & \quad x^2 \cdot x = x, \\
^{(3)} & \quad x \cdot xy = y, & ^{(4)} & \quad (x \cdot x)y = y, & ^{(5)} & \quad x \cdot y^2 = x, & ^{(6)} & \quad x \cdot x^2 = x, \\
^{(3)} & \quad xy \cdot y = x, & ^{(4)} & \quad (y \cdot x)y = x, & ^{(5)} & \quad x(y \cdot y) = x, & ^{(6)} & \quad x(x \cdot x) = x.
\end{align*}
\]

**Corollary 1.** All quasigroup identities of length 2 determine 14 different varieties distributing in 6 trusses according to the law of parastrophic symmetry. The trusses 1, 2 are totally-symmetric and the trusses 3, 4, 5, 6 are one-sided-symmetric.