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Some special \( p \)-groups and nearrings with identity

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Clearly every associative ring is a nearring and each group is the additive group of a nearring, but not necessarily of a nearring with identity. The question what group can be the additive group of a nearring with identity is far from solution.

We investigate \( p \)-groups with cyclic subgroup of index \( p \) as the additive groups of nearrings with identity.

In \([1\), Theorem 12.5.1\] it was proved that there exist seven types of \( p \)-groups with cyclic subgroup of index \( p \).

**Theorem 1.** Let \( G \) be a group from \([1\), Theorem 12.5.1\]. \( G \) is the additive group of a nearring with identity iff one of the following statement holds:

1. \( G = \langle a | a^p = 1 \rangle, \ n \geq 1 \).
2. \( G = \langle a, b | a^{p^n-1} = 1, b^p = 1, ba = ab \rangle, n \geq 2 \).
3. \( G = \langle a, b | a^{p^n-1} = 1, b^p = 1, ba = a^{1+p^n-2}b \rangle, p \) is odd, \( n \geq 3 \).
4. \( G \) is a dihedral group of order 8.
5. \( G = \langle a, b | a^{2^n-1} = 1, b^2 = 1, ba = a^{1+2^n-2}b \rangle, n \geq 4 \).

Denote by \( n(G) \) the number of all non-isomorphic zero-symmetric nearrings with identity whose additive group \( R^+ \) is isomorphic to the group \( G \).

So using \([3\), Theorem 7.1\] and \([2\) we can easily conclude the following result:

**Proposition 1.** Let \( G \) be a non-abelian group from Theorem 1. Then the following statements hold:

1. If \( p = 2 \) and \( n = 3 \), then \( n(G) = 7 \).
2. If \( p = 2 \) and \( n = 4 \), then \( n(G) = 32 \).
3. If \( p = 2 \) and \( n > 4 \), then \( n(G) = 2^{n+2} \).
4. If \( p = 3 \), then \( n(G) = 3^{n-2} \).
5. If \( p > 3 \), then \( n(G) = p^{n-3} \).

References

Reduction of nonsingular matrices over rings of almost stable range 1

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All rings consider of will be a commutative with nonzero units. Recall that a ring $R$ is a Bezout ring if it every finitely generated ideal is principal. A ring $R$ is called an elementary divisor ring if for any $n \times m$ matrix $A$ over $R$ there exist invertible matrices $P \in GL_n(R)$ and $Q \in GL_m(R)$ such that $PAQ = D$ is a diagonal matrix. $D = (d_{ii})$ and $d_{i+1,i+1}R \subseteq d_{ii}R$ [1].

We denote by $GE_n$ the subgroup of $GL_n(R)$ generated by the elementary matrices.

A ring $R$ is called a ring of stable range 1 if for any elements $a, b \in R$ the equality $aR + bR = R$ implies that there is some $x \in R$ such that $(a + bx)R = R$.

**Definition 1.** An element $a \neq 0$ of a commutative ring $R$ is called an element almost stable range 1 if the stable range of a factor-ring $R/aR$ is equal to 1. If all nonzero elements of a ring $R$ are elements of almost stable range 1 then we say that $R$ is a ring of almost stable range 1.

**Theorem 1.** Let $R$ be commutative Bezout domain of almost stable range 1, then for any nonsingular matrix of size $n$ we can find such unimodular matrices $P \in GE_n(R)$ and $Q \in GL_n(R)$, that

$$PAQ = \begin{pmatrix} \varepsilon_1 & 0 & \ldots & 0 \\ 0 & \varepsilon_2 & \ldots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \ldots & \varepsilon_n \end{pmatrix},$$

where $\varepsilon_i$ is divisor $\varepsilon_{i+1}$, $1 \leq i \leq n - 1$.

**References**