(xiii) $G = Q \times K$, where $K$ is a quasicyclic $p$-subgroup, $Q = C_G(Q)$ is an elementary abelian $q$-subgroup, $p$, $q$ are primes, $p \neq q$, $Q$ is a minimal normal subgroup of $G$.

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Semiscalar equivalence of one class of 3-by-3 matrices

Bohdan Shavarovskii

Let a matrix $F(x) \in M(3, \mathbb{C}[x])$ have a unit first invariant factor and only one characteristic root. We assume that this uniquely characteristic root is zero. In [1], the author proved that in the class $\{PF(x)Q(x)\}$, where $P \in GL(3, \mathbb{C})$, $Q(x) \in GL(3, \mathbb{C}[x])$ there exists a matrix

$$A(x) = \begin{pmatrix} 1 & 0 & 0 \\ a_1(x) & x^{k_1} & 0 \\ a_3(x) & a_2(x) & x^{k_2} \end{pmatrix}$$

(notation: $A(x) \approx F(x)$), which has the following properties:

(i) $\deg a_1 < k_1$, $\deg a_2$, $\deg a_3 < k_2$, $a_2(x) = x^{k_1}a'_2(x)$, $a_1(0) = a'_2(0) = a_3(0) = 0$;

(ii) $\co \deg a_3 \neq \co \deg a_1$, $\co \deg a'_2$, if $\co \deg a_3 < \co \deg a_2$;

(iii) $\co \deg a_3 \neq 2\co \deg a_1 + \co \deg a'_2$ and in $a_1(x)$ the monomial of the degree $2\co \deg a_1$ is absent, if $\co \deg a_3 \geq \co \deg a_2$.

Here $\co \deg$ denotes the junior degree of polynomial. The purpose of this report is to construct the canonical form of the matrix $F(x)$ in the class $\{PF(x)Q(x)\}$. If both elements $a_1(x)$, $a_2(x)$ of the matrix $A(x)$ are non-zero, then we may take their junior coefficients to be identity elements. In the opposite case, we may take the junior coefficients of the non-zero subdiagonal elements of the matrix $A(x)$ to be one. Such matrix $A(x)$ in [1] is called the reduced matrix. In this report we consider the case, when some of the elements $a_1(x)$, $a_2(x)$, $a_3(x)$ of the matrix $A(x)$ are equal to zero and at least one of them is different from zero.

**Theorem 1.** Let in the reduced matrix $A(x)$ the conditions $a_i(x) \neq 0$, for some index $i$ from set $\{1, 2, 3\}$ and $a_j(x) \equiv 0$ for the rest $j \in \{1, 2, 3\}$, $j \neq i$, be fulfilled. Then $A(x) \approx B(x)$, where in the reduced matrix

$$B(x) = \begin{pmatrix} 1 & 0 & 0 \\ b_1(x) & x^{k_1} & 0 \\ b_3(x) & b_2(x) & x^{k_2} \end{pmatrix},$$
the element $b_i(x) \neq 0$ does not contain $n_i$-monomial,
\[
    n_i = \begin{cases} 
        \text{deg} a_i, & i = 1, 3, \\
        \text{deg} a_i' + k_1, & i = 2, 
    \end{cases}
\]

$b_j(x) \equiv 0$. The matrix $B(x)$ is uniquely defined.

References

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On greatest common divisors and least common multiple of linear matrix equation solutions

Volodymyr Shchedryk

Investigation of linear equation solutions has a profound history. Due to applied and theoretical problems we need to find roots with certain predefined properties. Matrix equations were studied with a symmetry condition, with Hermitian positively defined condition, with minimal rank condition on the solutions.

Let $R$ be an associative ring with $1 \neq 0$. A set of all solutions of the equation $a = bx$ in $R$ is $c + Ann_r(b)$, where $c$ is some root one,
\[
    Ann_r(b) = \{ f \in R | bf = 0 \}.
\]

Such a description of the roots is not always convenient. We would like to have their image in the form of a product. In this connection, the question arises search for the generating element of this set.

Let $A, B$ be a matrices over ring $R$. If $A = BC$, then $A$ is a right multiple of $B$ and $B$ is a left divisor of $A$. If $A = DA_1$ and $B = DB_1$, then $D$ is a common left divisor of $A, B$; if, furthermore, $D$ is a right multiple of every common right divisor of $A$ and $B$, then $D$ is a left greatest common divisor of $A, B$.

If $M = NA = KB$, then $M$ is a common left multiple of $A$ and $B$, and; if, furthermore, $M$ is right divisor of every common left multiple of $A$ and $B$, then $M$ is a left least common multiple of $A$ and $B$. Greatest common left divisor and the least common right multiple of two given matrices over commutative elementary divisor domain are uniquely determined up to invertible right factors.

**Theorem 1.** Let $R$ be a commutative elementary divisor domain \cite{1}. Let an equation $A = BX$, where $A, B \in M_n(R)$ is solvable. Then the left greatest common divisor and the left least common multiple of its solutions are again its solution.

**Problem.** Describe a rings in which the sets of the roots of the linear equations contain a generating elements.