Let $\mathbb{Z}_{15}$ be a ring modulo 15, operations $(\ast), (\circ), (\cdot)$ defined on $\mathbb{Z}_{15}$ by the equalities
\[ x \ast y := 2x + 3 + 4y, \quad x \circ y := 4x + 3 + 2y, \quad x \cdot y := 8x + 6 - 2y \]
have left, right and middle inverse properties respectively and $\lambda(x) = 11x$, $\rho(x) = 11x$, $\mu(x) = 11x$.

References

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An invertibility criterion of composition of two multiary central quasigroups

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An $n$-ary operation $f$ defined on a set $Q$ is said to be invertible if it is invertible in each of the monoids $(O_n, +_i)$ of all $n$-ary operations defined on $Q$, where
\[ (f + g)(x_0, \ldots, x_{n-1}) := f(x_0, \ldots, x_{i-1}, g(x_0, \ldots, x_{n-1}), x_{i+1}, \ldots, x_{n-1}), \quad i = 0, \ldots, n-1. \]

An $n$-ary groupoid $(Q; f)$ is called: a quasigroup, if the operation is invertible and a group isotope, if there exists a group $(G; +)$ and bijections $\gamma_0, \ldots, \gamma_n$ from $Q$ to $G$ such that
\[ f(x_0, \ldots, x_{n-1}) = \gamma_n^{-1}(\gamma_0 x_0 + \ldots + \gamma_{n-1} x_{n-1}) \]
for all $x_0, \ldots, x_{n-1}$ in $Q$. It is easy to verify that a group isotope is a quasigroup. Let $0$ be an arbitrary element from $Q$, a sequence $(+, \alpha_0, \ldots, \alpha_{n-1}, a)$ is said to be a canonical decomposition (see [1]) of a group isotope $(Q; f)$ if $(Q; +, 0)$ is a group, $\alpha_0 0 = \ldots = \alpha_{n-1} 0 = 0$, $a \in Q$ and
\[ f(x_0, \ldots, x_{n-1}) = \alpha_0 x_0 + \ldots + \alpha_{n-1} x_{n-1} + a. \]

$(Q; +, 0)$ is called a canonical decomposition group and $\alpha_0, \ldots, \alpha_{n-1}$ are coefficients.

A group isotope is called central, if in a canonical decomposition the group is commutative and all coefficients are automorphisms of the group. A map $\alpha : A \to B$ is called ortho-complete,
if all preimages of elements from the set $B$ have the same cardinality. Thus, an $n$-ary operation $f$ defined on an $m$-element set $Q$ is ortho-complete if for any $a \in Q$ the equation $f(x_1, \ldots, x_n) = a$ has exactly $m^{n-1}$ solutions.

**Lemma 1.** [2] A finite $n$-ary quasigroup $(Q; f)$ is admissible iff for some $k \in \overline{1, n} := \{1, 2, \ldots, n\}$ there exists an $(n - 1)$-ary invertible operation $g$ such that the operation $h$ defined by

$$h(x_1, \ldots, x_{k-1}, x_{k+1}, \ldots, x_n) = f(x_1, \ldots, x_{k-1}, g(x_1, \ldots, x_{k-1}, x_{k+1}, \ldots, x_n), x_{k+1}, \ldots, x_n) \quad (1)$$

is ortho-complete.

**Theorem 1.** Let $(+, \alpha_0, \ldots, \alpha_{n-1}, a)$ and $(+, \beta_1, \ldots, \beta_{k-1}, \beta_{k+1}, \ldots, \beta_n, b)$ be canonical decompositions of central quasigroups $(Q; f)$ and $(Q; g)$ respectively. Then $(n - 1)$-ary operation $h$ defined by (1) is invertible iff for all $i \in \overline{1, n} \setminus \{k\}$ the endomorphism $\alpha_i + \alpha_k \beta_i$ of the group $(Q; +, 0)$ is its automorphism.

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**On total multiplication groups**

**Parascovia Syrbu**

The concept of multiplication group was introduced by Albert in the middle of the 20th century and at present is a standard tool in the algebraic quasigroup (loop) theory. Let $(Q, \cdot)$ be a quasigroup. The left, right and middle translations are denoted by $L_x$, $R_x$, $J_x$ respectively, and are defined as follows: $L_x(y) = x \cdot y$, $R_x(y) = y \cdot x$, $J_x(y) = x \cdot y$, $\forall x, y \in Q$. The groups $Mlt(Q) =$\(\langle L_x, R_x \mid x \in Q \rangle >\) and $TMlt(Q) =$\(\langle L_x, R_x, J_x \mid x \in Q >\)$ are called the multiplication group and the total multiplication group of $(Q, \cdot)$, respectively. If $(Q, \cdot)$ is a loop, then the stabilizer of its unit in $Mlt(Q)$ (resp. in $TMlt(Q)$) is called the inner mapping group (resp. the total inner mapping group) of $(Q, \cdot)$ and is denoted by $Inn(Q)$ (resp. $TInn(Q)$).

The total multiplication groups and total inner mapping groups have been considered at the end of 60s by Belousov in [1], where he noted that $TMlt(Q)$ is invariant under the parastrophy of quasigroups and gave a set of generators for the group $TInn(Q)$. Sets of generators of the total inner mapping group of a loop are also given by D. Stanovsky, P. Vojtechovsky [2], V. Shcherbacov [4] and P. Syrbu [3]. It is known that the total multiplication groups of isostrophic