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On induced modules over group rings of groups of finite rank

ANATOLII V. TUSEV

Let $G$ be a group and $k$ be a field. A $kG$-module $M$ is said to be imprimitive if there are a subgroup $H < G$ and a $kH$-submodule $N \leq M$ such that $M = N \otimes_{kH} kG$. If the module $M$ is not imprimitive then it is said to be primitive. A representation of the group $G$ is said to be primitive if the module of the representation is primitive.

Let $G$ be a group of finite rank $r(G)$ and $k$ be a field. A $kG$-module $M$ is said to be semi-imprimitive if there are subgroup $H < G$ and a $kH$-submodule $N \leq M$ such that $r(H) < r(G)$ and $M = N \otimes_{kH} kG$. If the module $M$ is not semi-imprimitive then it is said to be semi-primitive. A representation of the group $G$ is said to be semi-primitive if the module of the representation is semi-primitive. An element $g \in G$ (a subgroup $H \leq G$) is said to be orbital if $|G : C_G(g)| < \infty$ ($|G : N_G(H)| < \infty$). The set $\Delta(G)$ of all orbital elements of $G$ is a characteristic subgroup of $G$ which is said to be the $FC$-center of $G$.

In [1] Harper showed that any finitely generated not abelian-by-finite nilpotent group has an irreducible primitive representation over any not locally finite field. In [3] we proved that in the class of soluble groups of finite rank with the maximal condition for normal subgroups only polycyclic groups may have irreducible primitive faithful representations over a field of characteristic zero. In [2] Harper proved that if a polycyclic group $G$ has a faithful irreducible semi-primitive representation then $A \cap \Delta (G) \neq 1$ for any orbital subgroup $A$ of $G$. It is well known that any polycyclic group is linear and has finite rank.

**Theorem 1.** Let $G$ be a linear group of finite rank. Suppose that $G$ has a normal subgroup $1 \neq A$, such that $A \cap \Delta (G) = 1$. Let $k$ be a field of characteristic zero and let $M$ be an irreducible $kG$-module such that $C_G(M) = 1$. Then there are a subgroup $S \leq G$ and a $kS$-submodule $U \leq M$ such that $r(S) < (G)$ and $M = U \otimes_{kS} kG$.

**Corollary 1.** Let $G$ be a linear group of finite rank. If the group $G$ has a faithful irreducible semi-primitive representation over a field of characteristic zero then $A \cap \Delta (G) \neq 1$ for any orbital subgroup $A$ of $G$.

**References**

On irreducibility of monomial matrices of order 7 over local rings

ALEXANDER TYLYSHCHAK

The problem of classifying, up to similarity, all the matrices over a commutative ring (which is not a field) is usually very difficult; in most cases it is “unsolvable” (wild, as in the case of the rings of residue classes considered by Bondarenko [1]). In such situation, an important place is occupied by irreducible and indecomposable matrices over rings.

Let \( R \) be a commutative local ring with identity with Jacobson radical \( \text{Rad} R = tR, \ t \neq 0 \), \( n, k \) be a natural, \( 0 < k < n \),

\[
M(t, k, n) = \begin{pmatrix}
0 & \ldots & 0 & 0 & \ldots & 0 & t \\
1 & \ldots & 0 & \ldots & 0 & 0 & \ldots \\
\vdots & \ddots & \ddots & \ddots & \ddots & \ddots & \ddots \\
0 & \ldots & 1 & \ldots & 0 & 0 & \ldots \\
\vdots & \ldots & \ddots & \ddots & \ddots & \ddots & \ddots \\
0 & \ldots & 0 & \ldots & 0 & 0 & 0 \\
0 & \ldots & 0 & \ldots & t & 0 & 0
\end{pmatrix}
\]

be an \( n \times n \)-matrix. This matrices first arose in studying indecomposable representations of finite \( p \)-groups over commutative local rings [2].

The question when matrix \( M(t, k, n) \) is reducible had been solved, in particular, in following cases.

<table>
<thead>
<tr>
<th>( M(t, k, n) )</th>
<th>Case</th>
<th>Source</th>
</tr>
</thead>
<tbody>
<tr>
<td>irreducible</td>
<td>( k = 1, n - 1, \ t \neq 0 )</td>
<td>2</td>
</tr>
<tr>
<td>reducible</td>
<td>((k, n) &gt; 1)</td>
<td>3</td>
</tr>
<tr>
<td>irreducible</td>
<td>( n &lt; 7, \ (k, n) = 1, \ t \neq 0 )</td>
<td>4</td>
</tr>
<tr>
<td>reducible</td>
<td>( n = 7, \ k = 3, 4, \ t^2 = 0 )</td>
<td>[4, 5]</td>
</tr>
</tbody>
</table>

**Theorem 1.** Let \( n = 7, 0 < k < n, \ t^2 \neq 0 \). The matrix \( M(t, k, n) \) is irreducible over \( R \).

These studies were carried out together with V. M. Bondarenko.

**References**