Finite groups with given properties of normalizers of Sylow subgroups

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We consider only finite groups. We use notations and definitions from [1].

Let $F$ be a non-empty formation. A subgroup $H$ is called $F$-subnormal in $G$, if either $H = G$, or there exists a maximal chain of subgroups $H = H_0 \leq H_1 \leq \cdots \leq H_{n-1} \leq H_n = G$ such that $H_i \in F$ for $i = 1, \ldots, n$.

Recall that the class of groups $w^*F$ is defined as follows:

$$w^*F = \{ G \mid \pi(G) \subseteq \pi(F) \text{ and every normalizer of Sylow subgroup of } G \text{ is } F \text{-subnormal in } G \}.$$ 

**Theorem 1.** Let $F$ be a non-empty hereditary formation. Then the following statements are true.

1. $F \subseteq w^*F$.
2. $w^*F = w^*(w^*F)$.
3. If a formation $F_1 \subseteq F$ then $w^*F_1 \subseteq w^*F$.
4. $w^*F$ is a formation and from $G \in F$ it follows that every Hall subgroup of $G$ belongs to $F$.

According to [2], the arithmetic length of a soluble group $G$ is defined as $\max \{ l_p(G) \}$, where $l_p(G)$ is $p$-length of the group $G$ for all $p \in \pi(G)$. Note that the class $L_a(1)$ of all soluble groups whose arithmetic length $\leq 1$ is a hereditary saturated Fitting formation.

**Theorem 2.** Let $F$ be a hereditary saturated formation and $F \subseteq L_a(1)$. Then $w^*F = F$.

**Corollary 1.**

1. [3] If $N^2$ is the class of all metanilpotent groups, then $w^*N^2 = N^2$.
2. [3] If $N \mathfrak{A}$ is the class of all groups $G$ with the nilpotent commutator subgroup $G'$, then $w^*N \mathfrak{A} = N \mathfrak{A}$.
3. $w^*L_a(1) = L_a(1)$.

We note that $w^*N^3 \neq N^3$.

**References**

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**Cotransitive subsemigroups of the full transformation semigroup $T_n$**

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The concept of a cotransitive subsemigroup for transformations semigroups was introduced by R.P. Sullivan in the work [1]. It is used to describe the ideals. We restrict ourselves to the consideration of a full transformation semigroup $T_n$ of finite set $X$. For $\alpha \in T_n$ by $\pi_\alpha = \alpha \circ \alpha^{-1}$ we denote the partition of the set $X$ into equivalence classes. Let $ran \alpha = \{x_1, x_2, \ldots, x_k\} \subseteq X$, $A_i = \alpha^{-1}(x_i)$. Subsemigroup $S \subseteq T_n$ is called cotransitive, if for every $\alpha = (A_i \mid x_i) \in S$ with rank $k$ we have:

1. for every $\{b_1, b_2, \ldots, b_k\} \subseteq X$  
   $$\mu = \left( \begin{array}{c} A_i \\ b_i \end{array} \right) \in S;$$

2. for every $\{y_1, y_2, \ldots, y_k\} \subseteq X$ there exists $\lambda \in S$ such that $y_i \in \lambda^{-1}(x_i)$, $i = 1, k$.

If a cotransitive subsemigroup $S \subseteq T_n$ contains element of rank $k > 1$, then there exists such family of partitions $\{\pi_{\alpha} \mid \alpha \in S'\}$, $S' \subseteq S$ of a set $X$, that separates any its $k$ elements. For $k = 1$ there is the trivial partition $\rho(1)$ with one block.

Partitions $X = \bigcup_{i=1}^{k} A_i = \bigcup_{i=1}^{k} B_i$ are of the same type if sets ($|A_1|, |A_2|, \ldots, |A_k|$) and ($|B_1|$, $|B_2|, \ldots, |B_k|$) differ only in ordering. The partition $X = \bigcup_{i=1}^{k} A_i$ is called less than $X = \bigcup_{i=1}^{r} B_i$ if every block $B_i$ of the second partition is a union of several blocks of the first partition. We denote the lattice of all partitions of a set $X$ by $Part X$.

**Lemma 1.** Let $\{\rho_j(k)\}_{j \in J}$ is such family of partitions of a set $X$ into $k > 1$ blocks, that separates any its $k$ elements, $Q_k = \{\rho \in Part X \mid \rho_j(k) \leq \rho \text{ for some } j \in J\}$. Then for $k < n$

$$S = \{\alpha \in T_n \mid \pi_\alpha \in \bigcup_{i=1}^{k} Q_i\}$$

is cotransitive subsemigroup of semigroup $T_n$.

**Lemma 2.** Let $\mu_1, \mu_2, \ldots, \mu_m$ is a family of partitions of a set $X$ into $k$ blocks ($1 < k < n$), $\{\rho_j\}_{j \in J}$ is a family of all partitions of a set $X$, such that are of the same type with one of $\mu_i$, and