We shown that if \( X \) is not a Fischer class, then the conditions (2) and (3) theorem 1 are not true.

References

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On adjoint groups of radical rings

**Vladislav G. Yurashev**

An associative algebra \( R \) without identity is called radical if the set of its elements forms a group with respect to the operation \( a \circ b = a + b + ab \) and \( R \) is nilpotent if \( R^n = 0 \) for some positive integer \( n \). It is well-known that every nilpotent algebra is radical and the set of elements of \( R \) forms a group with respect to the operation \( a \circ b = a + b + ab \) with \( a, b \in R \). This group is called the adjoint group of \( R \) and is denoted by \( R^\circ \). Obviously any subalgebra of \( R \) is a subgroup of \( R^\circ \), but the converse is not true.

Radical algebras whose all subgroups of their adjoint groups are subalgebras were described in [1]. Recall also that a finite group \( G \) is said to be a Miller–Moreno group if \( G \) is non-abelian and all proper subgroups of \( G \) are abelian. The following assertion is proved in [2], Lemma 3.3.

**Lemma 1.** Let a Miller–Moreno \( p \)-group \( G \) be the adjoint group of a nilpotent \( p \)-algebra. Then one of the following statements holds:

1) \( G \) is a metacyclic \( 2 \)-group of order at most 16;
2) \( G \) is a non-metacyclic \( 2 \)-group of exponent 4 and of order at most 32;
3) \( G \) is a non-abelian \( p \)-group of order \( p^3 \) and exponent \( p \) for odd \( p \).

Using this lemma and the description of radical algebras given in [1], the following statement can be verified.

**Proposition 1.** If a Miller–Moreno \( p \)-group \( G \) is the adjoint group of a nilpotent algebra \( R \), then every subgroup of \( G \) is a subalgebra in \( R \).

It was proved in [3], Theorem 4.3, that every radical ring and in particular algebra whose adjoint group is generated by two elements is nilpotent. From this and Proposition 1 the following result is derived.

**Theorem 1.** Let \( R \) be a radical algebra over a field of prime characteristic \( p \). Then the following statements are equivalent:
1) every subgroup of the adjoint group \( R^\circ \) is a subalgebra in \( R \);
2) every abelian subgroup of the adjoint group \( R^\circ \) is a subalgebra in \( R \);
3) every non-abelian subgroup of the adjoint group $R^\circ$ is a subalgebra in $R$.

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Commutative Bezout ring, which is a ring of neat range 1

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All rings considered will be commutative with nonzero unit.

Recall that ring is Bezout ring if it finitely generated ideals is principal. Ring $R$ is said to have a stable range 2 if for every elements $a, b, c \in R$ such that $aR + bR + cR = R$ we have $(a + cx)R + (b + cy)R = R$ for some elements $x, y \in R$. Ring $R$ is called an elementary divisor ring if for any matrix $A$ of order $n \times m$ over $R$ there exist invertible matrices $P \in GL_n(R)$ and $Q \in GL_m(R)$ such that $PAQ = D$ is a diagonal matrix, $D = (d_{ii})$ and $d_{i+1,i+1}R \subset d_{ii}R$. A ring $R$ is called a clean ring if for any $a \in R$ there exist invertible element $u \in R$ and idempotent $e \in R$ such that $a = e + u$. Element $a \in R$ is called a neat element if factor-ring $R/aR$ is a clean ring. Ring $R$ is called a ring of neat range 1 if from condition $aR + bR = R$ implies that $a + bt$ is a neat element for some $t \in R$.

**Proposition 1.** Let $R$ be a commutative Bezout ring of neat range 1. Then for any ideal $I$ of $R$ factor-ring $R/I$ is a ring of neat range 1.

**Proposition 2.** A commutative Bezout ring is a ring of neat range 1 if and only if factor-ring $R/J(R)$ is a ring of neat range 1 (where $J(R)$ – is Jacobson radical).

**Theorem 1.** Commutative Bezout ring in which all zero divisors are in Jacobson radical is an elementary divisor ring if and only if it is a ring of neat range 1.

References


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