3) every non-abelian subgroup of the adjoint group \( R^o \) is a subalgebra in \( R \).

References


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Commutative Bezout ring, which is a ring of neat range 1

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All rings considered will be commutative with nonzero unit.

Recall that ring is Bezout ring if it finitely generated ideals is principal. Ring \( R \) is said to have a stable range 2 if for every elements \( a, b, c \in R \) such that \( aR + bR + cR = R \) we have \((a + cx)R + (b + cy)R = R \) for some elements \( x, y \in R \). Ring \( R \) is called an elementary divisor ring if for any matrix \( A \) of order \( n \times m \) over \( R \) there exist invertible matrices \( P \in GL_n(R) \) and \( Q \in GL_m(R) \) such that \( PAQ = D \) is a diagonal matrix, \( D = (d_{ii}) \) and \( d_{i+1,i+1} R \subset d_{ii} R \). A ring \( R \) is called a clean ring if for any \( a \in R \) there exist invertible element \( u \in R \) and idempotent \( e \in R \) such that \( a = e + u \). Element \( a \in R \) is called a neat element if factor-ring \( R/aR \) is a clean ring. Ring \( R \) is called a ring of neat range 1 if from condition \( aR + bR = R \) implies that \( a + bt \) is a neat element for some \( t \in R \).

PROPOSITION 1. Let \( R \) be a commutative Bezout ring of neat range 1. Then for any ideal \( I \) of \( R \) factor-ring \( R/I \) is a ring of neat range 1.

PROPOSITION 2. A commutative Bezout ring is a ring of neat range 1 if and only if factor-ring \( R/J(R) \) is a ring of neat range 1 (where \( J(R) \) – is Jacobson radical).

THEOREM 1. Commutative Bezout ring in which all zero divisors are in Jacobson radical is an elementary divisor ring if and only if it is a ring of neat range 1.

References


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J-Noetherian Bezout domain which are not of stable range 1

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All rings considered will be commutative and have identity.
A ring $R$ is a ring of stable range 1 if for any $a, b \in R$ such that $aR + bR = R$ we have $(a + bt)R = R$ for some $t \in R$.

An element $a$ is an element of stable range 1 if for any $b \in R$ such that $aR + bR = R$ we have $a + bt$ is an invertible element for some $t \in R$.

An element $a \in R$ is an element of almost stable range 1 if $R/\overline{a}R$ is a ring of stable range 1.

By a Bezout ring we mean a ring in which all finitely generated ideals are principal.

By a $J$-ideal of $R$ we mean an intersection of maximal ideals of $R$.

A ring $R$ is $J$-Noetherian provided $R$ has maximum condition of $J$-ideals.

A commutative ring $R$ is called an elementary divisor ring if for an arbitrary matrix $A$ of order $n \times m$ over $R$ there exist invertible matrices $P \in GL_n(R)$ and $Q \in GL_m(R)$ such that $PAQ = D$ is diagonal matrix, $D = (d_{ii})$, $d_{i+1,i+1}R \subset d_{ii}R$.

Let $R$ be a Bezout domain. An element $a \in R$ is called a neat element if for every elements $b, c \in R$ such that $bR + cR = R$ there exist $r, s \in R$ such that $a = rs$ where $rR + bR = R$, $sR + cR = R$ and $rR + sR = R$. A Bezout domain is said to be of neat range 1 if for any $c, b \in R$ such that $cR + bR = R$ there exists $t \in R$ such that $a + bt$ is a neat element.

**Theorem 1.** A commutative Bezout domain $R$ is an elementary divisor domain if and only if $R$ is a ring of neat range 1.

**Theorem 2.** A nonunit divisor of a neat element of a commutative Bezout domain is a neat element.

**Theorem 3.** Let $R$ be a $J$-Noetherian Bezout domain which is not a ring of stable range 1. Then in $R$ there exists an element $a \in R$ such that $R/aR$ is a local ring.

By [8], any adequate element of a commutative Bezout ring is a neat element. An element $a$ of a domain $R$ is said to be adequate, if for every element $b \in R$ there exist elements $r, s \in R$ such that (1) $a = rs$; (2) $rR + bR = R$ (3) $\overline{sR + bR} \neq R$ for any $\overline{s} \in R$ such that $sR \subset \overline{s}R \neq R$.

A domain $R$ is called adequate if every nonzero element of $R$ is adequate [4].

**Theorem 4.** Let $R$ be a commutative Bezout element and $a$ is non-zero nonunit element of $R$. If $R/aR$ is local ring, then $a$ is an adequate element.

**Theorem 5.** Let $R$ be a $J$-Noetherian Bezout domain which is not a ring of stable range 1. Then in $R$ there exists a nonunit adequate element.

**Theorem 6.** Let $R$ be a Bezout domain in which every nonzero nonunit element has only finitely many prime ideals minimal over it. Then the factor ring $R/aR$ is the finite direct sum of valuation rings.