Some homological property of simply connected bimodule problems with quasi multiplicative basis

Vyacheslav Babych, Nataliya Golovashchuk

Let $\mathcal{C}$ be the considered in [**1**] class of a faithful simply connected finite dimensional bimodule problems $\mathcal{A} = (\mathsf{K}, \mathsf{V})$ with nilpotent radical over an algebraically closed field $\Bbbk$ with a basic category K and a faithful finite dimensional K-bimodule V. Similarly to [**2, 3**], the quasi multiplicative basis Γ is constructed for such bimodule problem of bounded representative type.

According to [**1**], $\Gamma = \Gamma(\mathcal{A}) = (\Gamma_0, \Gamma_1 = \Gamma_1^0 \cup \Gamma_1^1, s, t)$ is a bigraph with a set of vertices Γ_0, a set of arrows Γ_1^i of degree $i \in \{0, 1\}$, and the maps $s, t : \Gamma_1 \to \Gamma_0$ matching an initial $s(a)$ and a terminal $t(a)$ vertex for any arrow $a \in \Gamma_1$.

Denote by $\mathbb{L} = \mathbb{L}(\Gamma) \simeq \mathbb{Z}^{|\Gamma_0|}$ a free lattice of the rank $|\Gamma_0|$, freely generated over $\mathbb{Z}$ by the system $\{e(i) \in \mathbb{Z}^{|\Gamma_0|} \mid i \in \Gamma_0\}$ such that $e(i)_j = \delta_{ij}$.

Given $x = \sum_{i\in\Gamma_0} x_i e(i)$, $y = \sum_{i\in\Gamma_0} y_i e(i) \in \mathbb{L}$ define the integer non symmetric bilinear form $\langle -, - \rangle : \mathbb{L} \times \mathbb{L} \longrightarrow \mathbb{Z}$ by setting $\langle x, y\rangle = \sum_{i\in\Gamma_0} x_i y_i - \sum_{a\in\Gamma_1^0} x_{s(a)} y_{t(a)} + \sum_{a\in\Gamma_1^1} x_{s(a)} y_{t(a)}$. The equality $\chi(x) = \langle x, x\rangle$ denotes the integer Tits quadratic form $\chi : \mathbb{L} \longrightarrow \mathbb{Z}$.

Denote by $\mathcal{R} = \mathcal{R}(\mathcal{A})$ the category of representations of bimodule problem $\mathcal{A}$, and let $\underline{\dim} X = \sum_{i\in\Gamma_0} \dim X_i e(i) \in \mathbb{L}$ be the dimension vector of $X \in \mathcal{R}(\mathcal{A})$. Then $\langle \underline{\dim} X, \underline{\dim} Y\rangle = \dim \mathrm{Hom}_\Bbbk(X, Y) - \dim \mathrm{Ext}^1_\Bbbk(X, Y)$. A representation $X \in \mathcal{R}(\mathcal{A})$ is called *brick* if $\mathrm{Hom}_\Bbbk(X, X) = \mathrm{End}_\Bbbk(X) = \Bbbk \cdot \mathbf{1}_X$. Thus a brick is indecomposable. If X is a brick then $\underline{\dim} X$ is a root of χ and $\mathrm{Ext}^1_\Bbbk(X, X) = 0$.

Theorem. *Let $\mathcal{A} \in \mathcal{C}$ be a simply connected bimodule problem having weakly positive Tits form χ. Then $\mathcal{A}$ is of finite representation type, every indecomposable representation is a brick, and for every pair $X_1, X_2 \in \mathcal{R}(\mathcal{A})$ of representations*

$$\dim \mathrm{Hom}(X_1, X_2) = \max\{\, 0, \quad \langle \underline{\dim} X_1, \underline{\dim} X_2 \rangle\},$$
$$\dim \mathrm{Ext}(X_1, X_2) = \max\{\, 0, -\langle \underline{\dim} X_1, \underline{\dim} X_2 \rangle\}.$$

In particular, $\dim \mathrm{Hom}(X_1, X_2) \cdot \dim \mathrm{Ext}(X_1, X_2) = 0$.

References

1. V. Babych, N. Golovashchuk, S. Ovsienko. *Generalized multiplicative bases for one-sided bimodule problems* // Algebra and Discrete Mathematics, Vol. **12** (2011), No. 2, 1 – 24.
2. *Bautista R., Gabriel P., Roiter A. V., Salmeron L.* Representation-finite algebras and multiplicative bases // Invent. math. – 1985. – **81** – p. 217–286.
3. *Golovashuk N. S.* Covering of a class of free matrix problems // Dokl. AN USSR, Ser. A. – 1987. – 10. – p. 3–5. (in russian)

Contact information

Vyacheslav Babych
Department of Mechanics and Mathematics, Taras Shevchenko National University of Kyiv, Kyiv, Ukraine
Email address: vyacheslav.babych@gmail.com
URL: https://sites.google.com/view/cgtds/

Nataliya Golovashchuk
Department of Mechanics and Mathematics, Taras Shevchenko National University of Kyiv, Kyiv, Ukraine
Email address: golovash@gmail.com

Key words and phrases. Bimodule problem, Tits form, representation type

The Generalized Weyl Poisson algebras and their Poisson simplicity criterion

Volodymyr Bavula

A new large class of Poisson algebras, the class of generalized Weyl Poisson algebras, is introduced. It can be seen as Poisson algebra analogue of generalized Weyl algebras. A Poisson simplicity criterion is given for generalized Weyl Poisson algebras and an explicit description of the Poisson centre is obtained. Many examples are considered.

References

1. V. V. Bavula, *The Generalized Weyl Poisson algebras ans their Poisson simplicity criterion*, arXiv:1902.00695.

Contact information

Volodymyr Bavula
School of Mathematics and Statistics, University of Sheffield, Sheffield, UK
Email address: v.bavula@sheffield.ac.uk

Key words and phrases. The Generalized Weyl Poisson algebras, a Poisson simplicity criterion, a Poisson ideal, the classical polynomial Poisson algebra

The specialized characters of the representation of the Lie algebra sl_3 in terms of q- and (q, p)-numbers

Leonid Bedratyuk, Ivan Kachuryk

Let Γ_λ be the standard irreducible complex representation of $\mathfrak{sl}_3$ with the highest weight $\lambda = (\lambda_1, \lambda_2) \in \mathbb{Z}^2$, $\dim \Gamma_\lambda = (\lambda_1 + 1)(\lambda_2 + 1)(\lambda_1 + \lambda_2)/2$.

Denote by Λ the weight lattice of all finite dimensional representation of $\mathfrak{sl}_3$, and let $\mathbb{Z}(\Lambda)$ be their group ring. The ring $\mathbb{Z}(\Lambda)$ is free $\mathbb{Z}$-module with the basis elements $e(\lambda)$, $\lambda = (\lambda_1, \lambda_2) \in \Lambda$, $e(\lambda)e(\mu) = e(\lambda + \mu)$, $e(0) = 1$. Let Λ_λ be the set of all weights of the representation Γ_λ. Then the formal character $\mathrm{Char}(\Gamma_\lambda)$ is defined as formal sum $\sum_{\mu \in \Lambda_\lambda} n_\lambda(\mu) e(\mu) \in \mathbb{Z}(\Lambda)$, here $n_\lambda(\mu)$ is the multiplicities of the weight μ in the representation Γ_λ. By replacing $e(m, n) := q^n p^m$ we obtain the specialized expression for the character of $\mathrm{Char}(\Gamma_{(n,m)}) \equiv [n, m]_{q,p}$.

We establish several relations between the specialized characters $[n, m]_{qp}$ and the quantum (q, p)-numbers

$$[r]_{q,p} = \frac{q^r - p^{-r}}{q - p^{-1}},$$

and in some cases between different types of q-numbers.