non-equivalent representations of the group $S_3$ of the degree not greater then 3:

$$\Gamma_1 : a \to 1,\ b \to 1; \quad \Gamma_2 : a \to -1,\ b \to 1; \quad \Gamma_3 : a \to \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix},\ b \to \begin{pmatrix} 0 & -1 \\ 1 & -1 \end{pmatrix};$$

$$\Gamma_4 : a \to \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \quad b \to \begin{pmatrix} -1 & -1 \\ 1 & 0 \end{pmatrix}; \quad \Gamma_5 : a \to \begin{pmatrix} 1 & 1 \\ 0 & -1 \end{pmatrix}, \quad b \to \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix};$$

$$\Gamma_6 : a \to \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix},\ b \to \begin{pmatrix} 1 & 0 & 1 \\ 0 & 0 & -1 \\ 0 & 1 & -1 \end{pmatrix};$$

$$\Gamma_7 : a \to \begin{pmatrix} -1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix},\ b \to \begin{pmatrix} 1 & 1 & 0 \\ 0 & -1 & -1 \\ 0 & -1 & 0 \end{pmatrix}.$$

Theorem 1. *Let $\Delta$ and $\Theta$ be an indecomposable integral representations of the group $S_3$. The tensor product $\Delta \otimes \Theta$ of the representations $\Delta$ and $\Theta$ is indecomposable if and only if one of the following conditions holds:*

1) *one of the representations $\Delta$ and $\Theta$ has degree 1;*
2) *both of the representations $\Delta$ and $\Theta$ are irreducible;*
3) *one of the representations $\Delta$ and $\Theta$ has degree 2 and another has degree 3.*

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# Diagonability of idempotent matrices over duo rings

Andriy Bilous

It is proved that a idempotent matrix over PT duo ring R is diagonalizable under a similarity transformation.

Definition 1. A ring $R$ is said to be a duo ring if every its left or right ideal is two sided.

Theorem 1. *Let $R$ be a duo ring and $A$ be an $n \times n$ idempotent matrix over $R$. If there exist invertible matrices $P$ and $Q$ such that $PAQ$ is a diagonal matrix, then there is an invertible matrix $U$ such that $UAU^{-1}$ is a diagonal matrix.*

Definition 2. A ring $R$ is a *PT* (projective trivial) ring if every idempotent matrix over $R$ is similar to a diagonal matrix.

Theorem 2. *Let $R$ be a PT ring. Then any unimodular vector $(a_1, a_2, \cdots, a_n)$ in $R^n$ is completable (i.e. can be seen as the first row of some invertible matrix).*

Definition 3. The row vector X is said to be a characteristic vector of $A \in R^n$ corresponding to $r \in R$ provided $(1)X$ is a basal vector and $(2)XA = rX$.

Theorem 3. *The following are equivalent for a duo ring $R$:*

*(1) Each idempotent matrix over $R$ is diagonalizable under a similarity transformation.*
*(2) Each idempotent matrix over $R$ has a characteristic vector.*

Theorem 4. *Let $R$ be an PT duo ring and $A$ be an $n \times n$ idempotent matrix over $R$. Then*

*(1) There is an invertible matrix $P$ with $PAP^{-1} = diag(a_1, a_2, \cdots, a_n)$ where $a_i$ divides $a_{i+1}$ for $1 \le i \le n-1$.*
*(2) If $Q$ is another invertible matrix with $QAQ^{-1} = diag(b_1, b_2, \cdots, b_n)$ where $b_i$ divides $b_{i+1}$ for $1 \le i \le n-1$, then $b_i = a_i$ for $1 \le i \le n$.*

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# Algebraic questions about a FTL physics

Enzo Bonacci

The recent proposal of a negative mass fluid to explain both the dark matter and energy [**7**] has renovated the interest for cosmological solutions based upon non-ordinary masses. Challenging the Λ-CDM paradigm, some fringe models are grounded on hypothetical interactions with antimatter [**5**] whereas others suppose the influence of faster than light (FTL) imaginary mass ([**4, 6, 8**]).

More than a decade ago ([**1**] — [**3**]) we supplied an organic description of all the possible states (positive, negative and imaginary mass) subsequent to modified Lorentz's equations giving physical significance to the energetic condition $|E| < m_0c^2$. Namely, we assumed that a fermion could pass from negative energy (identified as antimatter) to positive levels (i.e., the ordinary matter) through the interval between $-m_0c^2$ and $+m_0c^2$ where it would behave like a luxon ($v = c$) or a tachyon ($v > c$) keeping its half-integer spin.

We wish to illustrate the algebraic questions behind a so formulated FTL physics, included a falsification test currently being assembled at CERN's Antiproton Decelerator.