THEOREM 2. *Let R be a PT ring. Then any unimodular vector $(a_1, a_2, \cdots, a_n)$ in R^n is completable (i.e. can be seen as the first row of some invertible matrix).*

DEFINITION 3. The row vector X is said to be a characteristic vector of $A \in R^n$ corresponding to $r \in R$ provided (1)X is a basal vector and (2)$XA = rX$.

THEOREM 3. *The following are equivalent for a duo ring R:*

(1) Each idempotent matrix over R is diagonalizable under a similarity transformation.
(2) Each idempotent matrix over R has a characteristic vector.

THEOREM 4. *Let R be an PT duo ring and A be an $n \times n$ idempotent matrix over R. Then*

(1) There is an invertible matrix P with $PAP^{-1} = diag(a_1, a_2, \cdots, a_n)$ where a_i divides a_{i+1} for $1 \leq i \leq n-1$.
(2) If Q is another invertible matrix with $QAQ^{-1} = diag(b_1, b_2, \cdots, b_n)$ where b_i divides b_{i+1} for $1 \leq i \leq n-1$, then $b_i = a_i$ for $1 \leq i \leq n$.

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Algebraic questions about a FTL physics

ENZO BONACCI

The recent proposal of a negative mass fluid to explain both the dark matter and energy [**7**] has renovated the interest for cosmological solutions based upon non-ordinary masses. Challenging the Λ-CDM paradigm, some fringe models are grounded on hypothetical interactions with antimatter [**5**] whereas others suppose the influence of faster than light (FTL) imaginary mass ([**4, 6, 8**]).

More than a decade ago ([**1**] — [**3**]) we supplied an organic description of all the possible states (positive, negative and imaginary mass) subsequent to modified Lorentz's equations giving physical significance to the energetic condition $|E| < m_0c^2$. Namely, we assumed that a fermion could pass from negative energy (identified as antimatter) to positive levels (i.e., the ordinary matter) through the interval between $-m_0c^2$ and $+m_0c^2$ where it would behave like a luxon ($v = c$) or a tachyon ($v > c$) keeping its half-integer spin.

We wish to illustrate the algebraic questions behind a so formulated FTL physics, included a falsification test currently being assembled at CERN's Antiproton Decelerator.

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The duality in the affine actions on trees

Ievgen Bondarenko

Every action on a tree given by a (finite) automaton has an associated dual action given by the dual automaton. In this talk I will consider the affine groups of subrings of a global function field, construct their actions on a regular tree, and describe the dual action. In particular, this gives a natural family of bireversible automata and square complexes with interesting properties coming from the affine groups of global function fields. The talk is based on a joint work in progress with Dmytro Savchuk.

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On the classification of the serial principal posets

Vitaliy M. Bondarenko, Maryna Styopochkina

A finite poset S is called principal if the quadratic Tits form $q_S(z) := z_0^2 + \sum_{i\in S} z_i^2 + \sum_{i<j,i,j\in S} z_i z_j - z_0 \sum_{i\in S} z_i$ of S is non-negative and $\operatorname{Ker} q_S(z) := \{t \mid q_S(t) = 0\}$ is an infinite cyclic group, i.e. $\operatorname{Ker} q_S(z) = t_0\mathbb{Z}$ for some $t_0 \neq 0$. We call a principal poset S serial if for any $m \in \mathbb{N}$, there is a principal poset $S(m) \supset S$ such that $|S(m) \setminus S| = m$.