On classification of matrix representations of monoids of the fourth order

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We describe canonical forms of the matrix representations of monoids of the fourth order over an arbitrary field and classify (up to equivalence) all their indecomposable representations. We also indicate criteria on representation type.

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Reducibility of canonical t-cyclic monomial matrices over commutative local rings

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We study canonical t-cyclic matrices over commutative local rings.

Let K be a commutative local ring with radical $R \neq 0$ and let $t \in R$ such that $t^m = 0$, $t^{m-1} \neq 0$.

A cyclic matrix of the form

$$A = M_t(\overline{a}) = \begin{pmatrix} 0 & \dots & 0 & a_n \\ a_1 & \dots & 0 & 0 \\ \vdots & \ddots & \vdots & \vdots \\ 0 & \dots & a_{n-1} & 0 \end{pmatrix},$$

is called canonical cyclic. The sequence $\overline{a} = (a_1, \dots, a_{n-1}, a_n)$ is called the defining sequence of A. If all elements a_i have the form t^{s_i} $(t \in K)$, where $s_i \geq 0$ $(i = 1, 2, \dots, n)$, the matrix A is called canonical t-cyclic [**3**].

Theorem 1. *Any canonical t-cyclic matrix over K with defining sequence containing subsequence $(t^i, t^{p+q}, t^j, 1)$, where $i + q \geq m$, $j + p \geq m$, is reducible.*

Corollary 1. *Any canonical t-cyclic matrix over K with defining sequence containing subsequence $(t^{m-1}, t^2, t^{m-1}, 1)$ is reducible.*

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Normal loop rings

Victor Bovdi

Let KG be the loop ring of a di-associative loop G over the associative and commutative ring K with unity, let σ be an antiautomorphism of order two of G and let $f : G \to U(K)$ be a homomorphism from G to $U(K)$. For an element $x = \sum_{g\in G} \alpha_g g \in KG$ we define

$$x^{\alpha(f,\sigma)} = \sum_{g\in G} \alpha_g f(g)\sigma(g).$$

The map $\alpha(f, \sigma) : x \mapsto x^{\alpha(f,\sigma)}$ is an involution of KG if and only if

$$g\sigma(g) \in Kerf = \{\ h \in G \mid f(h) = 1\ \} \qquad \text{for all} \quad g \in G.$$

A loop ring KG is called normal if $xx^{\alpha(f,\sigma)} = x^{\alpha(f,\sigma)}x$ for all $x \in KG$. The description of the classical normal group rings and twisted group rings were obtained in [**1, 4**] and [**2, 3**], respectively.

In my talk we discus the question when a loop ring KG is normal.

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