Rota-type operators on a commutative modular group algebra

Victor Bovdi, Vasyl Laver

Currently (for example, see [**1, 2, 3**]) the Rota-type operators on associative algebras are actively studied. Examples of such operators are the following:

- Rota-Baxter operator of length λ: $f(x)f(y) = f(xf(y) + f(x)y + \lambda xy)$;
- Reynolds operator: $f(x)f(y) = f(xf(y) + f(x)y - f(x)f(y))$;
- Nijenhuis operator: $f(x)f(y) = f(xf(y) + f(x)y - f(xy))$;
- Average operator: $f(x)f(y) = f(xf(y))$.

All such Rota-type operators were considered on algebras over the field of characteristic 0.

We present Rota-type operators on the group algebra $\mathbb{F}G$ of a finite abelian 2-group G over the field $\mathbb{F}$ of characteristic 2 and give some constructions of such operators for arbitrary characteristic $p \geq 2$ (see [**4**]). While solving this problem the GAP System of computational algebra [**5**] was actively used.

References

1. V. Gubarev, *Rota-Baxter operators on unital algebras*, arXiv: 1805.00723 (2019) 1–37.
2. I.A. Karimjanov, I. Kaygorodov, M. Ladra. *Rota-type operators on null-filiform associative algebras*, Linear Multilinear Algebra, **44** (2018) 1-15.
3. L. Guo, M. Rosenkranz, and S.H. Zheng, *Rota-Baxter operators on the polynomial algebras, integration and averaging operators*, Pacific J. Math., (2) 275 (2015) 481- 507.
4. V. Bovdi, V. Laver, *Rota-type operators on a commutative modular group algebra*, Submitted for publication. (2019) 1-18.
5. The GAP Group. *GAP – Groups, Algorithms, and Programming, Version 4.10.0*, 2018. url: https://www.gap-system.org

Contact information

Victor Bovdi
Department of Mathematical Sciences, United Arab Emirates University, Al Ain, United Arab Emirates
Email address: vbovdi@gmail.com
URL: https://cos.uaeu.ac.ae/en/profile.shtml?email=v.bodi@uaeu.ac.ae

Vasyl Laver
Department of Informative and Operating Systems and Technologies, Uzhhorod National University, Uzhhorod, Ukraine
Email address: v.laver@gmail.com

Key words and phrases. Rota-type operators, group algebra, Reynolds operatoe, Nijenhuis operator, Average operator

On Leibniz algebras with two types of subalgebras

Vasyl Chupordia

Let L be an algebra over a field F with the binary operations + and [,]. Then L is called a *Leibniz algebra* (more precisely a left Leibniz algebra) if it satisfies the (left) Leibniz identity $[[a, b], c] = [a, [b, c]] - [b, [a, c]]$, for all $a, b, c \in L$.

Leibniz algebras whose subalgebras are ideals were described in [**1**]. Let L be a Leibniz algebra and A be a subalgebra of L. There are two ideals connected with A: $A^L = \bigcap_{A\subseteq I\trianglelefteq L} I$ and $Core_L(A) = \sum_{A\supseteq I\trianglelefteq L} I$. The ideal A^L is least ideal of L including A. $Core_L(A)$ is the greatest ideal of L which is contained in A. A subalgebra A of L is called an *contraideal* of L, if $A^L = L$. A subalgebra A of L is called *core-free* in L if $Core_L(A) = \langle 0\rangle$. From the definition it follows that the contraideals and core-free subalgebras are natural antipodes to the concepts of ideals. Leibniz algebras whose subalgebras are either ideals or contraideals were described in [**2**]. It was considered the next natural case – Leibniz algebras whose subalgebras are either ideals or core-free.

The intersection of all non-zero ideals of L is called *monolith* of L and denote $Mon(L)$. If $Mon(L) \neq \langle 0\rangle$ then L is said to be *monolithic*.

Theorem 1. *Let L be a Leibniz algebra, whose subalgebras are either ideals or core-free. If L is not Lie algebra and not all subalgebras are ideals then L is monolithic and it has one of the following types*

(1) if $\zeta(L) \neq \langle 0\rangle$ then $Mon(L) = \zeta(L) = Fz$, $L = \sum_{i\in I} C_i + B$, where
- *(a) C_i – abelian core-free subalgebra and $\left(\sum_{i\in I} C_i + \zeta(L)\right)/\zeta(L)$ is abelian;*
- *(b) B is an ideal in L, $\zeta(L) \leq B$, $[b,b] \neq 0$, for all $b \in B \setminus (Leib(L) \cup \zeta(L))$;*
- *(c) $[B, C_i], [C_i, B] \leq \zeta(L)$ for all $i \in I$;*

(2) if $\zeta(L) = \langle 0\rangle$ then $Mon(L) = \gamma_3(L) \neq \langle 0\rangle$ – abelian ideal and L is metabelian.

References

1. L.A. Kurdachenko, N.N. Semko, I.Ya. Subbotin, *The Leibniz algebras whose subalgebras are ideals*, Open Math. **15** (2017), 92–100.
2. L.A. Kurdachenko, I.Ya. Subbotin, V.S. Yashchuk, *On some antipodes of the ideals in leibniz algebras*, Journal of Algebra and Its Applications (2019), doi: 10.1142/S0219498820501133.

Contact information

Vasyl Chupordia
Department of Geometry and Algebra, Oles Honchar Dnipro National University, Dnipro, Ukraine
Email address: vchupordia@gmail.com

Key words and phrases. Leibniz algebra, ideal, core-free subalgebra, monolith

Classification of finite semigroups for which the inverse monoid of local automorphisms is a Δ-semigroup

Volodymyr Derech

A local automorphism of the semigroup S is defined as an isomorphism between two subsemigroups of this semigroup. The set of all local automorphisms of the semigroup S with respect to the ordinary operation of composition of binary relations forms an inverse monoid of local automorphisms. We denote this monoid by $LAut(S)$. Next, a semigroup S is called congruence-permutable if $\xi \circ \eta = \eta \circ \xi$ for any pair of congruences ξ, η on S. A semigroup S is called a Δ-semigroup if the lattice of its congruences forms a chain relative to the inclusion. It is obvious that any Δ-semigroup is congruence-permutable. A semigroup each element of which is an idempotent is called a band. A semigroup S with zero is called a nilsemigroup if, for any $x \in S$, there exists a natural number n such that $x^n = 0$.