Leibniz algebras whose subalgebras are ideals were described in [**1**]. Let L be a Leibniz algebra and A be a subalgebra of L. There are two ideals connected with A: $A^L = \bigcap_{A\subseteq I\trianglelefteq L} I$ and $Core_L(A) = \sum_{A\supseteq I\trianglelefteq L} I$. The ideal A^L is least ideal of L including A. $Core_L(A)$ is the greatest ideal of L which is contained in A. A subalgebra A of L is called an *contraideal* of L, if $A^L = L$. A subalgebra A of L is called *core-free* in L if $Core_L(A) = \langle 0\rangle$. From the definition it follows that the contraideals and core-free subalgebras are natural antipodes to the concepts of ideals. Leibniz algebras whose subalgebras are either ideals or contraideals were described in [**2**]. It was considered the next natural case – Leibniz algebras whose subalgebras are either ideals or core-free.

The intersection of all non-zero ideals of L is called *monolith* of L and denote $Mon(L)$. If $Mon(L) \neq \langle 0\rangle$ then L is said to be *monolithic*.

Theorem 1. *Let L be a Leibniz algebra, whose subalgebras are either ideals or core-free. If L is not Lie algebra and not all subalgebras are ideals then L is monolithic and it has one of the following types*

(1) if $\zeta(L) \neq \langle 0\rangle$ then $Mon(L) = \zeta(L) = Fz$, $L = \sum_{i\in I} C_i + B$, where
- *(a) C_i – abelian core-free subalgebra and $\left(\sum_{i\in I} C_i + \zeta(L)\right)/\zeta(L)$ is abelian;*
- *(b) B is an ideal in L, $\zeta(L) \leq B$, $[b, b] \neq 0$, for all $b \in B \setminus (Leib(L) \cup \zeta(L))$;*
- *(c) $[B, C_i], [C_i, B] \leq \zeta(L)$ for all $i \in I$;*

(2) if $\zeta(L) = \langle 0\rangle$ then $Mon(L) = \gamma_3(L) \neq \langle 0\rangle$ – abelian ideal and L is metabelian.

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Classification of finite semigroups for which the inverse monoid of local automorphisms is a Δ-semigroup

Volodymyr Derech

A local automorphism of the semigroup S is defined as an isomorphism between two subsemigroups of this semigroup. The set of all local automorphisms of the semigroup S with respect to the ordinary operation of composition of binary relations forms an inverse monoid of local automorphisms. We denote this monoid by $LAut(S)$. Next, a semigroup S is called congruence-permutable if $\xi \circ \eta = \eta \circ \xi$ for any pair of congruences ξ, η on S. A semigroup S is called a Δ-semigroup if the lattice of its congruences forms a chain relative to the inclusion. It is obvious that any Δ-semigroup is congruence-permutable. A semigroup each element of which is an idempotent is called a band. A semigroup S with zero is called a nilsemigroup if, for any $x \in S$, there exists a natural number n such that $x^n = 0$.

Theorem 1 (see [**1**], proposition 3). *Let S be a finite semigroup. If the inverse monoid of local automorphisms $LAut(S)$ is a congruence-permutable, then the semigroup S is either a group or a nilsemigroup, or a band.*

Theorem 2. *Let S be a finite band or a finite nilsemigroup. The following statements are equivalent:*

(a) *$LAut(S)$ is a congruence-permutable inverse semigroup;*
(b) *$LAut(S)$ is a Δ-semigroup.*

The following theorem was proved in [**2**].

Theorem 3. *Let S be a finite band. The inverse monoid $LAut(S)$ is a congruence-permutable if and only if S is:*

(1) *either a linearly ordered semilattice;*
(2) *or a primitive semilattice;*
(3) *or a semigroup of right zeros;*
(4) *or a semigroup of left zeros.*

A finite nilsemigroups for which the inverse monoid of local automorphisms is a congruence-permutable semigroup describe in [**3**].

Theorem 4. *Let G be a finite group. The inverse monoid $LAut(G)$ is a Δ-semigroup if and only if G is:*

(1) *either a group of prime order p, where $p - 1 = 2^k$ for some nonnegative integer k;*
(2) *or an elementary Abelian 2-group of order 2^n, where $n \geq 2$.*

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On conditions for the Brandt semigroup to be non isomorphic to the variant

Oleksandra Desiateryk

Proposition 1. *Let a variant $(S, *_a)$ be isomorphic to the Brandt semigroup. Then the semigroup S is 0-simple.*

Since we are interested in semigroups isomorphic to Brandt semigroup let us further consider the S as a 0-simple semigroup.