Theorem 1 (see [**1**], proposition 3). *Let S be a finite semigroup. If the inverse monoid of local automorphisms $LAut(S)$ is a congruence-permutable, then the semigroup S is either a group or a nilsemigroup, or a band.*

Theorem 2. *Let S be a finite band or a finite nilsemigroup. The following statements are equivalent:*

(a) *$LAut(S)$ is a congruence-permutable inverse semigroup;*
(b) *$LAut(S)$ is a Δ-semigroup.*

The following theorem was proved in [**2**].

Theorem 3. *Let S be a finite band. The inverse monoid $LAut(S)$ is a congruence-permutable if and only if S is:*

(1) *either a linearly ordered semilattice;*
(2) *or a primitive semilattice;*
(3) *or a semigroup of right zeros;*
(4) *or a semigroup of left zeros.*

A finite nilsemigroups for which the inverse monoid of local automorphisms is a congruence-permutable semigroup describe in [**3**].

Theorem 4. *Let G be a finite group. The inverse monoid $LAut(G)$ is a Δ-semigroup if and only if G is:*

(1) *either a group of prime order p, where $p - 1 = 2^k$ for some nonnegative integer k;*
(2) *or an elementary Abelian 2-group of order 2^n, where $n \geq 2$.*

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On conditions for the Brandt semigroup to be non isomorphic to the variant

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Proposition 1. *Let a variant $(S, *_a)$ be isomorphic to the Brandt semigroup. Then the semigroup S is 0-simple.*

Since we are interested in semigroups isomorphic to Brandt semigroup let us further consider the S as a 0-simple semigroup.

Proposition 2. *Let a variant $(S, *_a)$ be isomorphic to the finite Brandt semigroup. Then S is finite complete 0-simple semigroup.*

From the [**1**] we have that if a variant $(S, *_a)$ is 0-simple, then S is 0-simple. In the [**2**] we can find that a semigroup S is complete 0-simple if and only if the semigroup S does not contain bicyclic semigroup.

Let us consider a variant $(S, *_a)$ isomorphic to the finite Brandt semigroup. Since by the proposition 2 the semigroup S is finite complete 0-simple. Then let us consider more general case when the semigroup S is complete 0-simple. Then by the Rees theorem [**3**] a semigroup S is isomorphic to a Rees matrix semigroup over the group with zero $\mathcal{M}^0(G^0; I, J; P)$. Then $(S, *_a) \cong (\mathcal{M}^0(G^0; I, J; P), *_{A_{ij}})$ The next proposition is obvious.

Proposition 3. *A variant of the semigroup $\mathcal{M}^0(G^0; I, J; P)$ generated by any non zero Rees matrix A_{ij} is a Rees matrix semigroup with sandwich matrix $Q = P \cdot A_{ij} \cdot P$.*

Proposition 4. *Let matrix Q have a zero on lk position then all k column or l row is zero, or in the same time k column and l row.*

We proved the next important proposition.

Proposition 5. *Any variant $(\mathcal{M}^0(G^0; I, J; P), *_{A_{ij}})$ of Rees matrix semigroup is not isomorphic to Rees matrix semigroup with unit sandwich matrix $\mathcal{M}^0((G')^0; K, K; \triangle)$.*

Theorem 1. *Let semigroup S does not contain bicyclic subsemigroup and $a \in S$, then $(S, *_a)$ is not a Brandt semigroup.*

Since a finite semigroup does not contain a bicyclic semigroup we have the next corollary.

Corollary 1. *Finite Brand semigroup is not a variant of any semigroup.*

For the semigroup which has a bicyclic subsemigroup we have solved the case when sandwich element belongs to the bicyclic subsemigroup.

Theorem 2. *Let a semigroup S contain subsemigroup $\mathfrak{Bi}$, and $a \in \mathfrak{Bi}$. Then the variant $(S, *_a)$ is not a Brandt semigroup.*

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Quasigroups with some Bol-Moufang type identities

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Groupoid $(Q, *)$ is called a quasigroup, if the following conditions are true [**1**]: $(\forall u, v \in Q)(\exists!\, x, y \in Q)(u * x = v \,\&\, y * u = v)$.