Proposition 2. *Let a variant $(S, *_a)$ be isomorphic to the finite Brandt semigroup. Then $S$ is finite complete 0-simple semigroup.*

From the [**1**] we have that if a variant $(S, *_a)$ is 0-simple, then $S$ is 0-simple. In the [**2**] we can find that a semigroup $S$ is complete 0-simple if and only if the semigroup $S$ does not contain bicyclic semigroup.

Let us consider a variant $(S, *_a)$ isomorphic to the finite Brandt semigroup. Since by the proposition 2 the semigroup $S$ is finite complete 0-simple. Then let us consider more general case when the semigroup $S$ is complete 0-simple. Then by the Rees theorem [**3**] a semigroup $S$ is isomorphic to a Rees matrix semigroup over the group with zero $\mathcal{M}^0(G^0; I, J; P)$. Then $(S, *_a) \cong (\mathcal{M}^0(G^0; I, J; P), *_{A_{ij}})$ The next proposition is obvious.

Proposition 3. *A variant of the semigroup $\mathcal{M}^0(G^0; I, J; P)$ generated by any non zero Rees matrix $A_{ij}$ is a Rees matrix semigroup with sandwich matrix $Q = P \cdot A_{ij} \cdot P$.*

Proposition 4. *Let matrix $Q$ have a zero on $lk$ position then all $k$ column or $l$ row is zero, or in the same time $k$ column and $l$ row.*

We proved the next important proposition.

Proposition 5. *Any variant $(\mathcal{M}^0(G^0; I, J; P), *_{A_{ij}})$ of Rees matrix semigroup is not isomorphic to Rees matrix semigroup with unit sandwich matrix $\mathcal{M}^0((G')^0; K, K; \triangle)$.*

Theorem 1. *Let semigroup $S$ does not contain bicyclic subsemigroup and $a \in S$, then $(S, *_a)$ is not a Brandt semigroup.*

Since a finite semigroup does not contain a bicyclic semigroup we have the next corollary.

Corollary 1. *Finite Brand semigroup is not a variant of any semigroup.*

For the semigroup which has a bicyclic subsemigroup we have solved the case when sandwich element belongs to the bicyclic subsemigroup.

Theorem 2. *Let a semigroup $S$ contain subsemigroup $\mathfrak{Bi}$, and $a \in \mathfrak{Bi}$. Then the variant $(S, *_a)$ is not a Brandt semigroup.*

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*Key words and phrases.* Brandt semigroup, variant, sandwich semigroup, bicyclic semigroup.

# Quasigroups with some Bol-Moufang type identities

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Groupoid $(Q, *)$ is called a quasigroup, if the following conditions are true [**1**]: $(\forall u, v \in Q)(\exists!\, x, y \in Q)(u * x = v \,\&\, y * u = v)$.

We research the existence of left and right identity elements (i.e., left and right unit) in quasigroups with Bol-Moufang type identities which are listed in classical Fenvesh' article [**2**]. Numeration of identities is taken from [**2, 3**].

Theorem 1. *Quasigroup* $(Q, \cdot)$ *with any from identities* $F_1$, $F_3$, $F_5$, $F_{10}$, $F_{11}$, $F_{14}$, $F_{18}$, $F_{20}$, $F_{21}$, $F_{24}$, $F_{25}$, $F_{28}$, $F_{31}$, $F_{32}$, $F_{33}$, $F_{34}$, $F_{47}$, $F_{50}$, $F_{55}$, $F_{58}$ *is a group.*

We notice, formulated theorem is connected with the following Belousov's Problem # 18 [**1**].

*From what identities, that are true in a quasigroup* $Q(\cdot)$*, does it follow that the quasigroup* $Q(\cdot)$ *is a loop? (An example of such identity is the identity of associativity).*

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*Key words and phrases.* Quasigroup, group, unit, Bol-Moufang identity

# Morita equivalence of non-commutative schemes

Yuriy Drozd

A *non-commutative scheme* $\mathbb{X}$ is, by definition, a pair $(X, \mathcal{O}_{\mathbb{X}})$, where $X$ is a scheme and $\mathcal{O}_{\mathbb{X}}$ is a sheaf of $\mathcal{O}_X$-algebras which is quasi-coherent as a $\mathcal{O}_X$-module. We denote by $\mathsf{Qcoh}\,\mathbb{X}$ the category of quasi-coherent $\mathcal{O}_{\mathbb{X}}$-modules and by $\mathsf{Coh}\,\mathbb{X}$ the category of coherent $\mathcal{O}_{\mathbb{X}}$-modules. We call the non-commutative scheme $\mathbb{X}$ *noetherian* if $X$ is a noetherian scheme and $\mathcal{O}_{\mathbb{X}}$ is coherent as an $\mathcal{O}_X$-module.

A quasi-coherent $\mathcal{O}_{\mathbb{X}}$-module $\mathcal{P}$ is said to be

- *locally projective* if every point $x \in X$ has an affine open neighbourhood $U$ such that $\mathcal{P}(U)$ is a projective $\mathcal{O}_{\mathbb{X}}(U)$-module.
- *local generator* if every point $x \in X$ has an affine open neighbourhood $U$ such that for some $n$ there is an epimorphism of modules $n\mathcal{P}(U) \to \mathcal{O}_{\mathbb{X}}(U)$.
- *local progenerator* if it is a locally projective local generator.

Theorem. *Let* $\mathbb{X} = (X, \mathcal{O}_{\mathbb{X}})$ *and* $\mathbb{Y} = (Y, \mathcal{O}_{\mathbb{Y}})$ *be noetherian non-commutative schemes.*

(1) *Let* $f : X \to Y$ *be an isomorphism of schemes and* $\mathcal{P} \in \mathsf{Coh}\,\mathbb{X}$ *be a local progenerator such that* $\mathcal{E}nd_{\mathcal{O}_{\mathbb{X}}}\mathcal{P} \simeq (f^*\mathcal{O}_{\mathbb{Y}})^{\mathrm{op}}$*. Then the functor* $\Phi_{\mathcal{P}} : \mathsf{Qcoh}\,\mathbb{X} \to \mathsf{Qcoh}\,\mathbb{Y}$ *such that* $\Phi_{\mathcal{P}}\mathcal{F} = f_*\mathcal{H}om_{\mathcal{O}_{\mathbb{X}}}(\mathcal{P}, \mathcal{F})$ *is an equivalence.*

(2) *On the contrary, if* $\Phi : \mathsf{Qcoh}\,\mathbb{X} \to \mathsf{Qcoh}\,\mathbb{Y}$ *is an equivalence of categories, there is a unique isomorphism* $f : X \to Y$ *and a unique (up to isomorphism) local progenerator* $\mathcal{P} \in \mathsf{Coh}\,\mathbb{X}$ *such that* $\mathcal{E}nd_{\mathcal{O}_{\mathbb{X}}}\mathcal{P} \simeq (f^*\mathcal{O}_{\mathbb{Y}})^{\mathrm{op}}$ *and* $\Phi \simeq \Phi_{\mathcal{P}}$*.*