We research the existence of left and right identity elements (i.e., left and right unit) in quasigroups with Bol-Moufang type identities which are listed in classical Fenvesh' article [**2**]. Numeration of identities is taken from [**2, 3**].

Theorem 1. *Quasigroup* $(Q, \cdot)$ *with any from identities* $F_1$, $F_3$, $F_5$, $F_{10}$, $F_{11}$, $F_{14}$, $F_{18}$, $F_{20}$, $F_{21}$, $F_{24}$, $F_{25}$, $F_{28}$, $F_{31}$, $F_{32}$, $F_{33}$, $F_{34}$, $F_{47}$, $F_{50}$, $F_{55}$, $F_{58}$ *is a group.*

We notice, formulated theorem is connected with the following Belousov's Problem # 18 [**1**].

*From what identities, that are true in a quasigroup* $Q(\cdot)$, *does it follow that the quasigroup* $Q(\cdot)$ *is a loop? (An example of such identity is the identity of associativity).*

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# Morita equivalence of non-commutative schemes

Yuriy Drozd

A *non-commutative scheme* $\mathbb{X}$ is, by definition, a pair $(X, \mathcal{O}_{\mathbb{X}})$, where $X$ is a scheme and $\mathcal{O}_{\mathbb{X}}$ is a sheaf of $\mathcal{O}_X$-algebras which is quasi-coherent as a $\mathcal{O}_X$-module. We denote by $\mathsf{Qcoh}\,\mathbb{X}$ the category of quasi-coherent $\mathcal{O}_{\mathbb{X}}$-modules and by $\mathsf{Coh}\,\mathbb{X}$ the category of coherent $\mathcal{O}_{\mathbb{X}}$-modules. We call the non-commutative scheme $\mathbb{X}$ *noetherian* if $X$ is a noetherian scheme and $\mathcal{O}_{\mathbb{X}}$ is coherent as an $\mathcal{O}_X$-module.

A quasi-coherent $\mathcal{O}_{\mathbb{X}}$-module $\mathcal{P}$ is said to be

- *locally projective* if every point $x \in X$ has an affine open neighbourhood $U$ such that $\mathcal{P}(U)$ is a projective $\mathcal{O}_{\mathbb{X}}(U)$-module.
- *local generator* if every point $x \in X$ has an affine open neighbourhood $U$ such that for some $n$ there is an epimorphism of modules $n\mathcal{P}(U) \to \mathcal{O}_{\mathbb{X}}(U)$.
- *local progenerator* if it is a locally projective local generator.

Theorem. *Let* $\mathbb{X} = (X, \mathcal{O}_{\mathbb{X}})$ *and* $\mathbb{Y} = (Y, \mathcal{O}_{\mathbb{Y}})$ *be noetherian non-commutative schemes.*

*(1) Let* $f : X \to Y$ *be an isomorphism of schemes and* $\mathcal{P} \in \mathsf{Coh}\,\mathbb{X}$ *be a local progenerator such that* $\mathcal{E}nd_{\mathcal{O}_{\mathbb{X}}}\mathcal{P} \simeq (f^*\mathcal{O}_{\mathbb{Y}})^{\mathrm{op}}$. *Then the functor* $\Phi_{\mathcal{P}} : \mathsf{Qcoh}\,\mathbb{X} \to \mathsf{Qcoh}\,\mathbb{Y}$ *such that* $\Phi_{\mathcal{P}}\mathcal{F} = f_*\mathcal{H}om_{\mathcal{O}_{\mathbb{X}}}(\mathcal{P}, \mathcal{F})$ *is an equivalence.*

*(2) On the contrary, if* $\Phi : \mathsf{Qcoh}\,\mathbb{X} \to \mathsf{Qcoh}\,\mathbb{Y}$ *is an equivalence of categories, there is a unique isomorphism* $f : X \to Y$ *and a unique (up to isomorphism) local progenerator* $\mathcal{P} \in \mathsf{Coh}\,\mathbb{X}$ *such that* $\mathcal{E}nd_{\mathcal{O}_{\mathbb{X}}}\mathcal{P} \simeq (f^*\mathcal{O}_{\mathbb{Y}})^{\mathrm{op}}$ *and* $\Phi \simeq \Phi_{\mathcal{P}}$.

As the functor $\Phi_{\mathcal{P}}$ maps coherent modules to coherent ones and any equivalence of the categories $\mathsf{Coh}\,\mathbb{X} \to \mathsf{Coh}\,\mathbb{Y}$ uniquely extends to an equivalence $\mathsf{Qcoh}\,\mathbb{X} \to \mathsf{Qcoh}\,\mathbb{Y}$, the theorem remains valid if we replace the categories of quasi-coherent modules to those of coherent modules.

The proof is based on the "usual" Morita Theorem and the results of P. Gabriel [**1**].

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It is a joint work with Igor Burban.

# Solutions of the Sylvester matrix equation with triangular coefficients

Nataliia Dzhaliuk

Matrix Sylvester-type equations over different domains appear in various branches of mathematics, for example, in control theory and dynamical systems theory. The solvability of linear matrix equation

$$AX + YB = C \tag{1}$$

over a field and over a ring of polynomials was examined by Roth [**1**]: *Matrix equation* (1) *is solvable if and only if matrices* $M = \begin{bmatrix} A & C \\ 0 & B \end{bmatrix}$ *and* $N = \begin{bmatrix} A & 0 \\ 0 & B \end{bmatrix}$ *are equivalent, i.e., there exist invertible matrices* $U$ *and* $V$ *such that*

$$UMV = N. \tag{2}$$

Many authors extended the Roth theorem to the case of principal ideal rings, arbitrary commutative rings and other rings.

We consider a matrix equation (1) with triangular coefficients $A, B$ and $C \in M(n, R)$, i.e.,

$$A = \begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ 0 & a_{22} & \cdots & a_{2n} \\ \cdots & \cdots & \cdots & \cdots \\ 0 & 0 & \cdots & a_{nn} \end{bmatrix}, \; B = \begin{bmatrix} b_{11} & b_{12} & \cdots & b_{1n} \\ 0 & b_{22} & \cdots & b_{2n} \\ \cdots & \cdots & \cdots & \cdots \\ 0 & 0 & \cdots & b_{nn} \end{bmatrix}, \; C = \begin{bmatrix} c_{11} & c_{12} & \cdots & c_{1n} \\ 0 & c_{22} & \cdots & c_{2n} \\ \cdots & \cdots & \cdots & \cdots \\ 0 & 0 & \cdots & c_{nn} \end{bmatrix}$$

over commutative principal ideal ring $R$. Tian [**2**] presented a necessary and sufficient condition for the matrix equation $BXC = A$ over an arbitrary field to have a triangular block solution in terms of ranks and column spaces of this matrix equation's coefficients. We establish necessary and sufficient conditions for existence of triangular solutions for matrix equation (1) in term of elements of its matrix coefficients $A, B$ and $C$.

Theorem 1. *Let the matrix equation* (1) *with triangular coefficients* $A, B$ *and* $C \in M(n, R)$ *be solvable. The same triangular form's solutions for this equation exist if and only if the greatest common divisor of* $a_{ii}$ *and* $b_{ii}$ *is a divisor of* $c_{ii}$ *(i.e.,* $(a_{ii}, b_{ii}) | c_{ii}$*) for all* $i = 1, 2, \ldots, n$.