As the functor $\Phi_{\mathcal{P}}$ maps coherent modules to coherent ones and any equivalence of the categories $\mathsf{Coh}\,\mathbb{X} \to \mathsf{Coh}\,\mathbb{Y}$ uniquely extends to an equivalence $\mathsf{Qcoh}\,\mathbb{X} \to \mathsf{Qcoh}\,\mathbb{Y}$, the theorem remains valid if we replace the categories of quasi-coherent modules to those of coherent modules.

The proof is based on the "usual" Morita Theorem and the results of P. Gabriel [**1**].

### References

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It is a joint work with Igor Burban.

# Solutions of the Sylvester matrix equation with triangular coefficients

Nataliia Dzhaliuk

Matrix Sylvester-type equations over different domains appear in various branches of mathematics, for example, in control theory and dynamical systems theory. The solvability of linear matrix equation

$$AX + YB = C \tag{1}$$

over a field and over a ring of polynomials was examined by Roth [**1**]: *Matrix equation* (1) *is solvable if and only if matrices* $M = \begin{bmatrix} A & C \\ 0 & B \end{bmatrix}$ *and* $N = \begin{bmatrix} A & 0 \\ 0 & B \end{bmatrix}$ *are equivalent, i.e., there exist invertible matrices* $U$ *and* $V$ *such that*

$$UMV = N. \tag{2}$$

Many authors extended the Roth theorem to the case of principal ideal rings, arbitrary commutative rings and other rings.

We consider a matrix equation (1) with triangular coefficients $A, B$ and $C \in M(n, R)$, i.e.,

$$A = \begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ 0 & a_{22} & \cdots & a_{2n} \\ \cdots & \cdots & \cdots & \cdots \\ 0 & 0 & \cdots & a_{nn} \end{bmatrix},\ B = \begin{bmatrix} b_{11} & b_{12} & \cdots & b_{1n} \\ 0 & b_{22} & \cdots & b_{2n} \\ \cdots & \cdots & \cdots & \cdots \\ 0 & 0 & \cdots & b_{nn} \end{bmatrix},\ C = \begin{bmatrix} c_{11} & c_{12} & \cdots & c_{1n} \\ 0 & c_{22} & \cdots & c_{2n} \\ \cdots & \cdots & \cdots & \cdots \\ 0 & 0 & \cdots & c_{nn} \end{bmatrix}$$

over commutative principal ideal ring $R$. Tian [**2**] presented a necessary and sufficient condition for the matrix equation $BXC = A$ over an arbitrary field to have a triangular block solution in terms of ranks and column spaces of this matrix equation's coefficients. We establish necessary and sufficient conditions for existence of triangular solutions for matrix equation (1) in term of elements of its matrix coefficients $A, B$ and $C$.

Theorem 1. *Let the matrix equation* (1) *with triangular coefficients* $A, B$ *and* $C \in M(n, R)$ *be solvable. The same triangular form's solutions for this equation exist if and only if the greatest common divisor of* $a_{ii}$ *and* $b_{ii}$ *is a divisor of* $c_{ii}$ *(i.e.,* $(a_{ii}, b_{ii}) | c_{ii}$*) for all* $i = 1, 2, \ldots, n$.

Corollary 1. *Transforming matrices $U$ and $V$ from (2) have the following upper unitriangular form*

$$U = \begin{bmatrix} I & -Y \\ 0 & I \end{bmatrix}, V = \begin{bmatrix} I & -X \\ 0 & I \end{bmatrix},$$

*where matrices $X$ and $Y$ have the same triangular form as matrices $A, B$ and $C$ if and only if $(a_{ii}, b_{ii})|c_{ii}$ for all $i = 1, 2, \ldots, n$.*

## References

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# Some notes on orthogonality

Iryna Fryz

A tuple of $n$-ary operations $f_1, \ldots, f_k$ $(n \geq 2, k \leq n)$ defined on a set $Q$ $(m := |Q|)$ is called *orthogonal* [**1**], if for arbitrary $b_1, \ldots, b_k \in Q$ the system $\{f_i(x_1, \ldots, x_n) = b_i\}_{i=1}^{k}$ has exactly $m^{n-k}$ solutions.

Let $f$ be an $n$-ary operation on $Q$ and

$$\delta := \{i_1, \ldots, i_k\} \subset \overline{1, n} := \{1, \ldots, n\}, \quad \{j_1, \ldots, j_{n-k}\} := \overline{1, n} \setminus \delta, \quad \bar{a} := (a_{j_1}, \ldots, a_{j_{n-k}}).$$

An operation $f_{(\bar{a},\delta)}$ which is defined by

$$f_{(\bar{a},\delta)}(x_{i_1}, \ldots, x_{i_k}) := f(y_1, \ldots, y_n),$$

where $y_i := \begin{cases} x_i, & \text{if } i \in \delta, \\ a_i, & \text{if } i \notin \delta, \end{cases}$ is called an $(\bar{a}, \delta)$-*retract* or a $\delta$-*retract* of $f$. Operations $f_{1;(\bar{a}_1,\delta)}, \ldots, f_{k;(\bar{a}_k,\delta)}$ are called *similar* $\delta$-retracts of $n$-ary operations $f_1, \ldots, f_k$, if $\bar{a}_1 = \cdots = \bar{a}_k$. A $k$-tuple of $n$-ary operations is called $\delta$-*retractly orthogonal* [**4**], if all tuples of similar $\delta$-retracts of these operations are orthogonal.

The notion of perpendicularity of the maximal type from [**3**] can be defined using the definition of retract orthogonality: $n$-ary operations $g$ and $h$ are called *perpendicular of the type* $(\iota, \iota; m)$, if they are $\delta$-retractly orthogonal for all $\delta$ such that $|\delta| = 2$ i $m \in \delta$. The results from [**5**] imply the following statement.

Proposition 1. *If $n$-ary operations $g$ and $h$ are perpendicular of the type $(\iota, \iota; m)$, $m \in \overline{1, n}$, then they are $\delta$-retractly orthogonal for all $\delta \subset \overline{1, n}$, where $|\delta| > 1$ and $m \in \delta$.*

The relationships between retract orthogonality and strong orthogonality was described by G.B. Belyavskaya and G.L. Mullen [**2**] and the relationships between retract orthogonality and orthogonality was studied in [**5**].