Proposition 2. *Let g and h be n-ary quasigroups. The following statements are equivalent:*
(1) g and h are strongly orthogonal;
(2) g and h are perpendicular of the type $(\iota,\iota;m)$ for all $m\in\overline{1,n}$;
(3) g and h are δ-retractly orthogonal for all $\delta\subset\overline{1,n}$;
(4) for an arbitrary $m\in\overline{1,n}$ operation $g\underset{m}{\oplus}h$ is invertible, where

$$(g\underset{m}{\oplus}h)(x_1,\dots,x_n):=g(x_1,\dots,x_{m-1},h(x_1,\dots,x_n),x_{m+1},\dots,x_n).$$

References

1. G. Belyavskaya, G.L. Mullen, *Orthogonal hypercubes and n-ary operations*, Quasigroups Related Systems **13** (2005), no. 1, 73–86.
2. G. Belyavskaya, G.L. Mullen, *Strongly orthogonal and uniformly orthogonal many-placed operations*, Algebra Discrete Math. **5** (2006), no. 1, 1–17.
3. F.M. Sokhatsky, I.V. Fryz, *Invertibility criterion of composition of two multiary quasigroups*, Comment. Math. Univ. Carolin. **53** (2012), no. 3, 429–445.
4. I.V. Fryz, F.M. Sokhatsky, *Block composition algorithm for constructing orthogonal n-ary operations*, Discrete Math. **340** (2017), iss. 8, 1957-1966.
5. I.V. Fryz, *Orthogonality and retract orthogonality of operations*, Bul. Acad. Ştiinţe Repub. Mold. Mat. **1(86)** (2018), 24–33.

Contact information

Iryna Fryz
Vasyl' Stus Donetsk National University, Vinnytsia, Ukraine
Email address: iryna.fryz@ukr.net

Key words and phrases. Orthogonality, perpendicularity, strong orthogonality

Diagonal reduction of matrices over commutative semihereditary Bezout rings

Andrii Gatalevych

All rings considered will be commutative and have identity. Recently there has been some interest in the polynomial ring $R[x]$, where R is a von Neumann regular ring. Such a ring is a Bezout ring, semihereditary ring, and so Hermite ring. Thus, it is natural to ask whether or not $R[x]$ is an elementary divisor ring. This question is answered affirmative in [3]. It is an open problem whether or not every Bezout domain is an elementary divisor ring and more generally: whether or not every semihereditary Bezout ring is an elementary divisor ring.

We obtain a complete characterization of semihereditary elementary divisor ring through its homomorphic images.

Mc Adam S. and Swan R. G. studied comaximal factorization in commutative rings [2]. Following them, we give the following definitions.

Definition 1. A nonzero element a of a ring R is called inpseudo-irreducible if for any representation $a=b\cdot c$ we have $bR+cR=R$.

Definition 2. An element a of a ring R is called pseudo-irreducible if for any representation $a=b\cdot c$, where $b,c\notin U(R)$, we have $bR+cR\neq R$.

Other definitions can be found in the articles [1, 4].

Theorem 1. *Let R be a Bezout ring of stable range 2. A regular element $a\in R$ is inpseudo-irreducible iff R/aR is a von Neumann regular ring.*

Theorem 2. *Let R be a Bezout ring of stable range 2. A regular element $a \in R$ is pseudo-irreducible iff R/aR is an indecomposable ring.*

Theorem 3. *Let R be a Bezout ring of stable range 2. A regular element $a \in R$ is an adequate element iff R/aR is a semiregular ring.*

Theorem 4. *Let R be a Bezout ring of stable range 2 and of Gelfand range 1. Then R is an elementary divisor ring.*

Theorem 5. *Let R be a Bezout domain. Then the following statements are equivalent*
1) R is an elementary divisor ring.
2) R is a ring of Gelfand range 1.

Theorem 6. *Let R be a semihereditary PM Bezout ring. Then R is an elementary divisor ring.*

References

1. I. Kaplansky, *Elementary divisors and modules*, Trans. Amer. Math. Soc. **66** (1949), 464–491.
2. McAdam, S., Swan, R. G., *Unique comaximal factorization*, J. Algebra. **276** (2004), 180–192.
3. Shores T., *Modules over semihereditary Bezout rings*, Proc. Amer. Math. Soc. **46** (1974), 211–213.
4. B.V. Zabavsky, *Conditions for stable range of an elementary divisor rings*, Comm. Algebra **45** (2017), no. 9, 4062–4066.

Contact information

Andrii Gatalevych
Ivan Franko National University of Lviv, Lviv, Ukraine
Email address: gatalevych@ukr.net
URL: http://www.lnu.edu.ua/

Key words and phrases. Bezout ring, elementary divisor ring, semihereditary ring, stable range, Gelfand range 1, adequate element, Gelfand element

Bezout rings with nonzero principal Jacobson radical

Andrii Gatalevych, Anatolii Dmytruk

All rings considered are commutative with $1 \neq 0$. Let us consider the example of M. Henriksen $R = \{z_0 + a_1x + a_2x + \ \dots \mid z_0 \in Z,\ a_i \in Q\}$ [**1**]. It has been constructed as an example of a commutative Bezout domain, which is an elementary divisor ring and is not an adequate ring. We note that its Jacobson radical is a nonzero prime ideal, which is not a principal ideal and stable range of the ring R equals 2. The issue arises about the structure of a Bezout domain in which a Jacobson radical is a nonzero principal ideal.

Definition 1. A ring R is called a Bezout ring if its every finitely generated ideal is principal.

Definition 2. A ring R is called a ring of stable range 1, if for any $a,\ b \in R$ such that $aR + bR = R$, there exists such an element $y \in R$ that $(a + by)R = R$ [**2**].

Theorem 1. *Let R be a commutative Bezout domain in which a Jacobson radical $J(R)$ is a nonzero principal ideal. Then R is a ring of stable range 1.*

Theorem 2. *Let R be a commutative Bezout domain, and let for the element $a \in R\backslash\{0\}$, a Jacobson radical of the factor ring $J({}^{R}/_{aR})$ is a nonzero principal ideal. Then the element a is contained only in the finite number of maximal ideals that are principal.*