Theorem 2. *Let R be a Bezout ring of stable range 2. A regular element $a \in R$ is pseudo-irreducible iff R/aR is an indecomposable ring.*

Theorem 3. *Let R be a Bezout ring of stable range 2. A regular element $a \in R$ is an adequate element iff R/aR is a semiregular ring.*

Theorem 4. *Let R be a Bezout ring of stable range 2 and of Gelfand range 1. Then R is an elementary divisor ring.*

Theorem 5. *Let R be a Bezout domain. Then the following statements are equivalent*
1) R is an elementary divisor ring.
2) R is a ring of Gelfand range 1.

Theorem 6. *Let R be a semihereditary PM Bezout ring. Then R is an elementary divisor ring.*

References

1. I. Kaplansky, *Elementary divisors and modules*, Trans. Amer. Math. Soc. **66** (1949), 464–491.
2. McAdam, S., Swan, R. G., *Unique comaximal factorization*, J. Algebra. **276** (2004), 180–192.
3. Shores T., *Modules over semihereditary Bezout rings*, Proc. Amer. Math. Soc. **46** (1974), 211–213.
4. B.V. Zabavsky, *Conditions for stable range of an elementary divisor rings*, Comm. Algebra **45** (2017), no. 9, 4062–4066.

Contact information

Andrii Gatalevych
Ivan Franko National University of Lviv, Lviv, Ukraine
Email address: gatalevych@ukr.net
URL: http://www.lnu.edu.ua/

Key words and phrases. Bezout ring, elementary divisor ring, semihereditary ring, stable range, Gelfand range 1, adequate element, Gelfand element

Bezout rings with nonzero principal Jacobson radical

Andrii Gatalevych, Anatolii Dmytruk

All rings considered are commutative with $1 \neq 0$. Let us consider the example of M. Henriksen $R = \{z_0 + a_1x + a_2x + \ \dots \mid z_0 \in Z,\ a_i \in Q\}$ [**1**]. It has been constructed as an example of a commutative Bezout domain, which is an elementary divisor ring and is not an adequate ring. We note that its Jacobson radical is a nonzero prime ideal, which is not a principal ideal and stable range of the ring R equals 2. The issue arises about the structure of a Bezout domain in which a Jacobson radical is a nonzero principal ideal.

Definition 1. A ring R is called a Bezout ring if its every finitely generated ideal is principal.

Definition 2. A ring R is called a ring of stable range 1, if for any $a,\ b \in R$ such that $aR + bR = R$, there exists such an element $y \in R$ that $(a + by)R = R$ [**2**].

Theorem 1. *Let R be a commutative Bezout domain in which a Jacobson radical $J(R)$ is a nonzero principal ideal. Then R is a ring of stable range 1.*

Theorem 2. *Let R be a commutative Bezout domain, and let for the element $a \in R\backslash\{0\}$, a Jacobson radical of the factor ring $J({}^{R}\!/_{aR})$ is a nonzero principal ideal. Then the element a is contained only in the finite number of maximal ideals that are principal.*

References

1. M. Henriksen, *Some remarks about elementary divisor rings*, Michigan Math. J. **3** (1955–1956), 159–163.
2. V. Zabavsky, *Reduction of matrices over Bezout rings of stable rank no more than 2*, Ukr. Math. J. **55** (2003), no. 4, 550–554.

Contact information

Andrii Gatalevych
Ivan Franko National University of Lviv, Lviv, Ukraine
Email address: gatalevych@ukr.net
URL: http://http://new.mmf.lnu.edu.ua/employee/hatalevych-a-i

Anatolii Dmytruk
Ivan Franko National University of Lviv, Lviv, Ukraine
Email address: tolikd1488@gmail.com
URL: http://http://new.mmf.lnu.edu.ua

Key words and phrases. Bezout ring, Jacobson radical, stable range 1

On automorphisms of superextensions of semigroups

Volodymyr Gavrylkiv

A family $\mathcal{M}$ of non-empty subsets of a set X is called *an upfamily* if for each set $A \in \mathcal{M}$ any subset $B \supset A$ of X belongs to $\mathcal{M}$. By $\upsilon(X)$ we denote the set of all upfamilies on a set X. Each family $\mathcal{B}$ of non-empty subsets of X generates the upfamily $\{A \subset X : \exists B \in \mathcal{B}\ (B \subset A)\}$ which we denote by $\langle B \subset X : B \in \mathcal{B}\rangle$. An upfamily $\mathcal{F}$ that is closed under taking finite intersections is called a *filter*. A filter $\mathcal{U}$ is called an *ultrafilter* if $\mathcal{U} = \mathcal{F}$ for any filter $\mathcal{F}$ containing $\mathcal{U}$. The family $\beta(X)$ of all ultrafilters on a set X is called the *Stone-Čech compactification* of X, see [**6**]. An ultrafilter $\langle\{x\}\rangle$, generated by a singleton $\{x\}$, $x \in X$, is called *principal*. Each point $x \in X$ is identified with the principal ultrafilter $\langle\{x\}\rangle$ generated by the singleton $\{x\}$, and hence we can consider $X \subset \beta(X) \subset \upsilon(X)$. It was shown in [**3**] that any associative binary operation $* : S \times S \to S$ can be extended to an associative binary operation $* : \upsilon(S) \times \upsilon(S) \to \upsilon(S)$ by the formula

$$\mathcal{L} * \mathcal{M} = \Big\langle \bigcup_{a \in L} a * M_a : L \in \mathcal{L},\ \ \{M_a\}_{a \in L} \subset \mathcal{M} \Big\rangle$$

for upfamilies $\mathcal{L}, \mathcal{M} \in \upsilon(S)$. In this case the Stone-Čech compactification $\beta(S)$ is a subsemigroup of the semigroup $\upsilon(S)$. The semigroup $\upsilon(S)$ contains as subsemigroups many other important extensions of S. In particular, it contains the semigroup $\lambda(S)$ of maximal linked upfamilies. An upfamily $\mathcal{L}$ of subsets of S is said to be *linked* if $A \cap B \neq \emptyset$ for all $A, B \in \mathcal{L}$. A linked upfamily $\mathcal{M}$ of subsets of S is *maximal linked* if $\mathcal{M}$ coincides with each linked upfamily $\mathcal{L}$ on S that contains $\mathcal{M}$. It follows that $\beta(S)$ is a subsemigroup of $\lambda(S)$. The space $\lambda(S)$ is well-known in General and Categorial Topology as the *superextension* of S, see [**7**].

Given a semigroup S we shall discuss the algebraic structure of the automorphism group $\mathrm{Aut}\,(\lambda(S))$ of the superextension $\lambda(S)$ of S. We show that any automorphism of a semigroup S can be extended to an automorphism of its superextension $\lambda(S)$, and the automorphism group $\mathrm{Aut}\,(\lambda(S))$ of the superextension $\lambda(S)$ of a semigroup S contains a subgroup, isomorphic to the group $\mathrm{Aut}\,(S)$. We describe in [**1, 2, 4**] automorphism groups of superextensions of groups,