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On automorphisms of superextensions of semigroups

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A family $\mathcal{M}$ of non-empty subsets of a set X is called *an upfamily* if for each set $A \in \mathcal{M}$ any subset $B \supset A$ of X belongs to $\mathcal{M}$. By $\upsilon(X)$ we denote the set of all upfamilies on a set X. Each family $\mathcal{B}$ of non-empty subsets of X generates the upfamily $\{A \subset X : \exists B \in \mathcal{B}\ (B \subset A)\}$ which we denote by $\langle B \subset X : B \in \mathcal{B}\rangle$. An upfamily $\mathcal{F}$ that is closed under taking finite intersections is called a *filter*. A filter $\mathcal{U}$ is called an *ultrafilter* if $\mathcal{U} = \mathcal{F}$ for any filter $\mathcal{F}$ containing $\mathcal{U}$. The family $\beta(X)$ of all ultrafilters on a set X is called the *Stone-Čech compactification* of X, see [**6**]. An ultrafilter $\langle\{x\}\rangle$, generated by a singleton $\{x\}$, $x \in X$, is called *principal*. Each point $x \in X$ is identified with the principal ultrafilter $\langle\{x\}\rangle$ generated by the singleton $\{x\}$, and hence we can consider $X \subset \beta(X) \subset \upsilon(X)$. It was shown in [**3**] that any associative binary operation $* : S \times S \to S$ can be extended to an associative binary operation $* : \upsilon(S) \times \upsilon(S) \to \upsilon(S)$ by the formula

$$\mathcal{L} * \mathcal{M} = \Big\langle \bigcup_{a \in L} a * M_a : L \in \mathcal{L},\ \ \{M_a\}_{a \in L} \subset \mathcal{M} \Big\rangle$$

for upfamilies $\mathcal{L}, \mathcal{M} \in \upsilon(S)$. In this case the Stone-Čech compactification $\beta(S)$ is a subsemigroup of the semigroup $\upsilon(S)$. The semigroup $\upsilon(S)$ contains as subsemigroups many other important extensions of S. In particular, it contains the semigroup $\lambda(S)$ of maximal linked upfamilies. An upfamily $\mathcal{L}$ of subsets of S is said to be *linked* if $A \cap B \neq \emptyset$ for all $A, B \in \mathcal{L}$. A linked upfamily $\mathcal{M}$ of subsets of S is *maximal linked* if $\mathcal{M}$ coincides with each linked upfamily $\mathcal{L}$ on S that contains $\mathcal{M}$. It follows that $\beta(S)$ is a subsemigroup of $\lambda(S)$. The space $\lambda(S)$ is well-known in General and Categorial Topology as the *superextension* of S, see [**7**].

Given a semigroup S we shall discuss the algebraic structure of the automorphism group $\mathrm{Aut}\,(\lambda(S))$ of the superextension $\lambda(S)$ of S. We show that any automorphism of a semigroup S can be extended to an automorphism of its superextension $\lambda(S)$, and the automorphism group $\mathrm{Aut}\,(\lambda(S))$ of the superextension $\lambda(S)$ of a semigroup S contains a subgroup, isomorphic to the group $\mathrm{Aut}\,(S)$. We describe in [**1, 2, 4**] automorphism groups of superextensions of groups, finite monogenic semigroups, null semigroups, almost null semigroups, right zero semigroups, left zero semigroups and all three-element semigroups.

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Invariant random subgroups of rank 1 Lie groups and hyperbolic groups and their growth rates

Ilya Gekhtman, Arie Levit

Invariant random subgroups (IRS) are conjugacy invariant probability measures on the space of subgroups of a given group G. They arise naturally as point stabilizers of probability measure preserving actions. The space of invariant random subgroups of SL_2R can be regarded as a natural compactification of the moduli space of Riemann surfaces, related to the Deligne-Mumford compactification. Invariant random subgroups can be regarded as a generalization both of normal subgroups and of lattices in topological groups. As such, it is interesting to extend results from the theories of normal subgroups and of lattices to the IRS setting.

Jointly with Arie Levit, we prove such a result: the critical exponent (exponential growth rate) of an infinite IRS in an isometry group of a Gromov hyperbolic space (such as a rank 1 Lie group, or a hyperbolic group) is almost surely greater than half the Hausdorff dimension of the boundary.

This generalizes an analogous result of Matsuzaki-Yabuki-Jaerisch for normal subgroups.

As a corollary, we obtain that if Γ is a typical subgroup and X a rank 1 symmetric space then $\lambda_0(X/\Gamma) < \lambda_0(X)$ where λ_0 is the bottom of the spectrum of the Laplacian. The proof uses ergodic theorems for actions of hyperbolic groups.

I will also talk about results about growth rates of normal subgroups of hyperbolic groups that inspired this work. Based mostly on the paper "Critical exponents of invariant random subgroups in negative curvature." (llya Gekhtman and Arie Levit, GAFA 2019)