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Geometry of numerical series and two-symbol systems of encoding of real numbers

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Let

$$\sum a_n = a_1 + a_2 + \ldots + a_n + r_n = s_n + r_n \tag{1}$$

be an absolutely convergent series. The number x defined by subset $M \subset N$ is called the incomplete sum or subsum of series (1) and is denoted by $x(M)$. It is evident that

$$x(M) = \sum_{n=1}^{\infty} \varepsilon_n a_n \equiv \Delta_{\varepsilon_1 \varepsilon_2 \ldots \varepsilon_n}, \quad \text{where} \quad \varepsilon_n = \begin{cases} 1 & \text{if } n \in M, \\ 0 & \text{if } n \notin M. \end{cases}$$

If M runs over the set of all subsets of the set of positive integers N, we have the numerical set E, i.e.,

$$E[a_n] = \{x : \ x(M), \ M \subset N\}.$$

This set is called the set of incomplete sums of series (1).

Now topological and metric types of the sets of incomplete sums of convergent series are well known. However, criteria for these sets to be nowhere dense sets, Cantorvals or sets of zero measure are not found yet.

Some series provide a basis for two-symbol systems of encoding (representation) for real numbers (with a zero or non-zero redundancy), but, for some other series, two-symbol alphabet is insufficient for creation of a numeral system and should be expanded. This is a topic of this talk. The main object is a polybase two-symbol numeral system with a zero redundancy such that one its base is positive and other one is negative ($0 < q_0 < 1$, $q_1 \equiv q_0 - 1$). We discuss some facts related to geometry of representation, normal properties of numbers in their representations as well as some algebraic aspects of this topic. In particular, we consider groups of transformations preserving tails of "representations", frequencies of digits, etc. Left and right shift operators, inversor of digits, various metrizations of the space of representations form a basis for ergodic theory corresponding to this numeral system. Problems of probabilistic number theory are also considered.

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On some combinatorial identities involving the Horadam numbers

Taras Goy

In [5, 6], Horadam defined generalized Fibonacci numbers $\{w_n(a,b;p,q)\}$, or briefly $\{w_n\}$, which satisfy the second-order homogeneous linear recurrence relation

$$w_n = pw_{n-1} - qw_{n-2}, \quad n \geq 2, \tag{1}$$

where $w_0 = a$, $w_1 = b$ and a, b, p, q are integers.

This sequence generalizes many number sequences, such as Fibonacci, Lucas, Pell, Jacobsthal sequences, among others.

We study some families of Toeplitz-Hessenberg determinants the entries of which are Horadam numbers. These determinant formulas may also be rewritten as identities involving sums of products of the Horadam numbers and multinomial coefficients.

Let $\varepsilon = a^2q - abp + b^2$, $|s| = s_1 + s_2 + \cdots + s_n$, $\sigma_n = s_1 + 2s_2 + \cdots + ns_n$, and $p_n(s) = \frac{(s_1+\cdots+s_n)!}{s_1!\cdots s_n!}$ denotes the multinomial coefficient.

Theorem 1. *For all $n \geq 2$, the following formulas fold*

$$\sum_{\sigma_n = n} (-1)^{|s|} p_n(s) \left(\frac{w_1}{w_0}\right)^{s_1} \left(\frac{w_2}{w_0}\right)^{s_2} \cdots \left(\frac{w_n}{w_0}\right)^{s_n} = \frac{\varepsilon(ap-b)^{n-2}}{a^n},$$

$$\sum_{\sigma_n = n} (-1)^{|s|} p_n(s) \left(\frac{w_2}{w_0}\right)^{s_1} \left(\frac{w_4}{w_0}\right)^{s_2} \cdots \left(\frac{w_{2n}}{w_0}\right)^{s_n} = \frac{\varepsilon p^2(ap^2-aq+bp)^{n-2}}{a^n},$$

$$\sum_{\sigma_n = n} (-1)^{|s|} p_n(s) \left(\frac{w_2}{w_1}\right)^{s_1} \left(\frac{w_3}{w_1}\right)^{s_2} \cdots \left(\frac{w_{n+1}}{w_1}\right)^{s_n} = \frac{\varepsilon a^{n-2}q^{n-1}}{b^n},$$

$$\sum_{\sigma_n = n} (-1)^{|s|} p_n(s) \left(\frac{w_3}{w_1}\right)^{s_1} \left(\frac{w_5}{w_1}\right)^{s_2} \cdots \left(\frac{w_{2n+1}}{w_1}\right)^{s_n} = \frac{\varepsilon q^{n-1}p^2(ap-b)^{n-2}}{b^n},$$

where the summation is over integers $s_i \geq 0$ satisfying $s_1 + 2s_2 + \cdots + ns_n = n$.

These identities generalize some identities which we have obtained in [1–4].

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