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On σ-nilpotency of finite groups

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All considered groups are finite and G always denotes a finite group. The symbol $\pi(G)$ denotes the set of all primes dividing the order of G. Two groups A and B are called *isoordic* if $|A| = |B|$.

Let σ be some partition of the set of all primes $\mathbb{P}$, that is, $\sigma = \{\sigma_i | i \in I\}$, where $\mathbb{P} = \bigcup_{i\in I} \sigma_i$ and $\sigma_i \cap \sigma_j = \emptyset$ for all $i \neq j$, and we put, following [**5**], $\sigma(G) = \{\sigma_i | \sigma_i \cap \pi(G) \neq \emptyset\}$. G is said to be: *σ-primary* [**5**] if G is a σ_i-group for some i; *σ-decomposable* (Shemetkov [**4**]) or *σ-nilpotent* (Guo and Skiba [**1**]) if $G = G_1 \times \cdots \times G_n$ for some σ-primary groups $G_1, \ldots, G_n$.

A subgroup A of G is called *σ-subnormal* in G [**5**] if it is *$\mathfrak{N}_\sigma$-subnormal in* G in the sense of Kegel [**2**], that is, there is a subgroup chain

$$A = A_0 \leq A_1 \leq \cdots \leq A_n = G$$

such that either $A_{i-1} \trianglelefteq A_i$ or $A_i/(A_{i-1})_{A_i}$ is σ-primary for all $i = 1, \ldots, n$. We use $i_\sigma(G)$ to denote the *number of classes of isoordic non-σ-subnormal subgroups* of G.

We study the structure of G depending on the invariant $i_\sigma(G)$. In particular, we obtained the conditions of σ-nilpotency of G with restrictions on $i_\sigma(G)$. For example, the following theorem was proved.

Theorem. [**3**, Theorem 1.7] *If $i_\sigma(G) \leq |\sigma(G)| - 2$, then G is σ-nilpotent.*

Note that Theorem is a corollary of the more general result [**3**, Theorem 1.2].

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Necessary and sufficient condition for the existence of one-point time on an oriented set

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Definition 1. The ordered pair $\mathcal{M} = \left(\mathfrak{Bs}(\mathcal{M}), \underset{\mathcal{M}}{\leftarrow}\right)$ is called an **oriented set** if and only if $\mathfrak{Bs}(\mathcal{M})$ is some non-empty set ($\mathfrak{Bs}(\mathcal{M}) \neq \emptyset$) and $\underset{\mathcal{M}}{\leftarrow}$ is arbitrary reflexive binary relation on $\mathfrak{Bs}(\mathcal{M})$. In this case the set $\mathfrak{Bs}(\mathcal{M})$ is named the **basic** set or the set of all **elementary states** of the oriented set $\mathcal{M}$ and the relation $\underset{\mathcal{M}}{\leftarrow}$ is named by the **directing relation of changes (transformations)** of $\mathcal{M}$.

In the case where the oriented set $\mathcal{M}$ is known in advance, the char $\mathcal{M}$ in the notation $\underset{\mathcal{M}}{\leftarrow}$ will be released, and we will use the notation $\leftarrow$ instead. From an intuitive point of view, oriented sets may be interpreted as the most primitive models of sets of evolving objects.

Definition 2. Let $\mathcal{M}$ be an oriented set and $\mathbb{T} = (\mathbf{T}, \leq)$ be a linearly ordered set. A mapping $\psi : \mathbf{T} \mapsto 2^{\mathfrak{Bs}(\mathcal{M})}$ is referred to as **time** on $\mathcal{M}$ if the following conditions are satisfied:

1. For any elementary state $x \in \mathfrak{Bs}(\mathcal{M})$ there exists an element $t \in \mathbf{T}$ such that $x \in \psi(t)$.

2. If $x_1, x_2 \in \mathfrak{Bs}(\mathcal{M})$, $x_2 \leftarrow x_1$ and $x_1 \neq x_2$, then there exist elements $t_1, t_2 \in \mathbf{T}$ such that $x_1 \in \psi(t_1)$, $x_2 \in \psi(t_2)$ and $t_1 < t_2$ (this means that there is a temporal separateness of successive unequal elementary states).

In this case the elements $t \in \mathbf{T}$ we call the **moments of time**.

It turns out that any oriented set $\mathcal{M}$ can be chronologized (that is we can define some time on it). To make sure this we may consider any linearly ordered set $\mathbb{T} = (\mathbf{T}, \leq)$, which contains at least two elements ($\mathbf{card}(\mathbf{T}) \geq 2$) and put, $\psi(t) := \mathfrak{Bs}(\mathcal{M})$, $t \in \mathbf{T}$.

Definition 3. Let $\mathcal{M}$ be an oriented set.

a. The time $\psi : \mathbf{T} \mapsto 2^{\mathfrak{Bs}(\mathcal{M})}$ is called by **quasi one-point** if for any $t \in \mathbf{T}$ the set $\psi(t)$ is a singleton.

b. The time ψ is called **one-point** if the following conditions are satisfied:
(a) the time ψ is quasi one-point;
(b) for every $x_1, x_2 \in \mathfrak{Bs}(\mathcal{M})$ the conditions $x_1 \in \psi(t_1)$, $x_2 \in \psi(t_2)$ and $t_1 \leq t_2$, assure the correlation $x_2 \leftarrow x_1$.

Example 1. Let us consider an arbitrary mapping $f : \mathbb{R} \mapsto \mathbb{R}^d$ ($d \in \mathbb{N}$). This mapping can be interpreted as equation of motion of a single material point in the space $\mathbb{R}^d$. The mapping f generates the oriented set $\mathcal{M}_f = \left(\mathfrak{Bs}\left(\mathcal{M}_f\right), \underset{\mathcal{M}_f}{\leftarrow}\right)$, where $\mathfrak{Bs}\left(\mathcal{M}_f\right) = \mathcal{R}(f) = \{f(t) \,|\, t \in \mathbb{R}\} \subseteq \mathbb{R}^d$ and for $x, y \in \mathfrak{Bs}(\mathcal{M})$ the correlation $y \underset{\mathcal{M}_f}{\leftarrow} x$ holds if and only if there exist $t_1, t_2 \in \mathbb{R}$ such,