Contact information

Viktoria A. Gritskova (Kovaleva)
Department of Mathematics and Technologies of Programming, Francisk Skorina Gomel State University, Gomel, Belarus
Email address: vika.kovalyova@rambler.ru

Key words and phrases. Finite group, isoordic groups, σ-nilpotent group, σ-subnormal subgroup.

Necessary and sufficient condition for the existence of one-point time on an oriented set

Yaroslav Ivanovich Grushka

Definition 1. The ordered pair $\mathcal{M}=\left(\mathfrak{Bs}(\mathcal{M}),\underset{\mathcal{M}}{\leftarrow}\right)$ is called an **oriented set** if and only if $\mathfrak{Bs}(\mathcal{M})$ is some non-empty set ($\mathfrak{Bs}(\mathcal{M})\neq\emptyset$) and $\underset{\mathcal{M}}{\leftarrow}$ is arbitrary reflexive binary relation on $\mathfrak{Bs}(\mathcal{M})$. In this case the set $\mathfrak{Bs}(\mathcal{M})$ is named the **basic** set or the set of all **elementary states** of the oriented set $\mathcal{M}$ and the relation $\underset{\mathcal{M}}{\leftarrow}$ is named by the **directing relation of changes (transformations)** of $\mathcal{M}$.

In the case where the oriented set $\mathcal{M}$ is known in advance, the char $\mathcal{M}$ in the notation $\underset{\mathcal{M}}{\leftarrow}$ will be released, and we will use the notation $\leftarrow$ instead. From an intuitive point of view, oriented sets may be interpreted as the most primitive models of sets of evolving objects.

Definition 2. Let $\mathcal{M}$ be an oriented set and $\mathbb{T}=(\mathbf{T},\leq)$ be a linearly ordered set. A mapping $\psi:\mathbf{T}\mapsto 2^{\mathfrak{Bs}(\mathcal{M})}$ is referred to as **time** on $\mathcal{M}$ if the following conditions are satisfied:

1. For any elementary state $x\in\mathfrak{Bs}(\mathcal{M})$ there exists an element $t\in\mathbf{T}$ such that $x\in\psi(t)$.

2. If $x_1,x_2\in\mathfrak{Bs}(\mathcal{M})$, $x_2\leftarrow x_1$ and $x_1\neq x_2$, then there exist elements $t_1,t_2\in\mathbf{T}$ such that $x_1\in\psi(t_1)$, $x_2\in\psi(t_2)$ and $t_1<t_2$ (this means that there is a temporal separateness of successive unequal elementary states).

In this case the elements $t\in\mathbf{T}$ we call the **moments of time**.

It turns out that any oriented set $\mathcal{M}$ can be chronologized (that is we can define some time on it). To make sure this we may consider any linearly ordered set $\mathbb{T}=(\mathbf{T},\leq)$, which contains at least two elements ($\mathbf{card}(\mathbf{T})\geq 2$) and put, $\psi(t):=\mathfrak{Bs}(\mathcal{M})$, $t\in\mathbf{T}$.

Definition 3. Let $\mathcal{M}$ be an oriented set.

a. The time $\psi:\mathbf{T}\mapsto 2^{\mathfrak{Bs}(\mathcal{M})}$ is called by **quasi one-point** if for any $t\in\mathbf{T}$ the set $\psi(t)$ is a singleton.

b. The time ψ is called **one-point** if the following conditions are satisfied:

(a) the time ψ is quasi one-point;

(b) for every $x_1,x_2\in\mathfrak{Bs}(\mathcal{M})$ the conditions $x_1\in\psi(t_1)$, $x_2\in\psi(t_2)$ and $t_1\leq t_2$, assure the correlation $x_2\leftarrow x_1$.

Example 1. Let us consider an arbitrary mapping $f:\mathbb{R}\mapsto\mathbb{R}^d$ ($d\in\mathbb{N}$). This mapping can be interpreted as equation of motion of a single material point in the space $\mathbb{R}^d$. The mapping f generates the oriented set $\mathcal{M}_f=\left(\mathfrak{Bs}\left(\mathcal{M}_f\right),\underset{\mathcal{M}_f}{\leftarrow}\right)$, where $\mathfrak{Bs}\left(\mathcal{M}_f\right)=\mathcal{R}(f)=\{f(t)\,|\,t\in\mathbb{R}\}\subseteq\mathbb{R}^d$ and for $x,y\in\mathfrak{Bs}(\mathcal{M})$ the correlation $y\underset{\mathcal{M}_f}{\leftarrow}x$ holds if and only if there exist $t_1,t_2\in\mathbb{R}$ such, that $x = f(t_1)$, $y = f(t_2)$ and $t_1 \leq t_2$. It is easy to verify, that the following mapping is a one-point time on $\mathcal{M}_f$:

$$\psi(t) = \{f(t)\} \subseteq \mathfrak{Bs}(\mathcal{M}), \quad t \in \mathbb{R}.$$

Example 1 makes clear the notion of one-point time. It is evident, that any one-point time is quasi one-point. There exist the counterexamples, which show that the inverse statement, in general, is not true.

THEOREM 1 (see [**1, 2**]). *Any oriented set $\mathcal{M}$ can be quasi one-point chronologized (this means that we can define some quasi one-point time on $\mathcal{M}$).*

On any oriented set $\mathcal{M}$ we introduce the following additional binary relation:

a. For every $x, y \in \mathfrak{Bs}(\mathcal{M})$ we note $y \overset{+}{\underset{\mathcal{M}}{\leftarrow}} x$ if and only if $y \underset{\mathcal{M}}{\leftarrow} x$ and $x \underset{\mathcal{M}}{\not\leftarrow} y$.

b. In the cases where it does not lead to misunderstanding we use the notation $y \overset{+}{\leftarrow} x$ instead of the record $y \overset{+}{\underset{\mathcal{M}}{\leftarrow}} x$.

DEFINITION 4. The oriented set $\mathcal{M}$ is called **quasi-chain** if and only if the following conditions are satisfied:

- For any $x_1, x_2 \in \mathfrak{Bs}(\mathcal{M})$ it holds at least one from the correlations $x_2 \leftarrow x_1$ or $x_1 \leftarrow x_2$.
- For every $x_0, x_1, x_2, x_3 \in \mathfrak{Bs}(\mathcal{M})$ the conditions $x_3 \overset{+}{\leftarrow} x_2$, $x_2 \leftarrow x_1$ and $x_1 \overset{+}{\leftarrow} x_0$ lead to the correlation $x_3 \overset{+}{\leftarrow} x_0$ (quasitransitivity).

It is easy to prove that the transitivity of the binary relation $\leftarrow$ on the oriented set $\mathcal{M}$ implies its quasitransitivity. It can be proven that the inverse statement in general is not valid. That is there exist the oriented set $\mathcal{M}$ such that the relation $\underset{\mathcal{M}}{\leftarrow}$ is quasitransitive but not transitive.

The main result of the talk is the following theorem, which gives the necessary and sufficient condition of existence for one-point time on the oriented set.

THEOREM 2 (ZF+AC). *The oriented $\mathcal{M}$ set can be one-point chronologized if and only if it is quasi-chain.*

We emphasize that proof of the necessity for Theorem 2 does not require the axiom of choice (AC). This axiom is needed only for the proof of sufficiency of the condition, pointed out in Theorem 2.

References

1. Grushka Ya.I., *Primitive changeable sets and their properties*, Mathematical Bulletin of the Shevchenko Scientific Society. **9** (2012), 52–80.
2. Grushka Ya.I., *Draft introduction to abstract kinematics. (Version 2.0)*, Preprint: ResearchGate, DOI: https://doi.org/10.13140/RG.2.2.28964.27521, 2017.

CONTACT INFORMATION

Yaroslav Ivanovich Grushka
Department of Nonlinear analysis, Institute of Mathematics NAS of Ukraine, Kyiv, Ukraine
Email address: grushka@imath.kiev.ua
URL: https://www.imath.kiev.ua/people/profile.php?pid=166&tab=1&lang=en

Key words and phrases. Oriented sets, changeable sets, time, ordered sets

This research was partially supported by Budget Program “Support to the development of priority areas of scientific research” KPKVK 6541230.