This theorem again show that the spectrum can not determine the structure of the Cayley graphs. The proof of the above results, in particular, uses the Hulanicki criterion of amenability of groups in terms of the weak containment of unitary representations, as well as its weak analogue for spectra of covering graphs obtained by A. Dudko and speaker.

Define a **spectral measure** by $\mu(B) = \langle E(B)\delta_e, \delta_e\rangle$ where $B \subset \mathbb{R}$ Borel subsets, $\{E(B)\}$ spectral projections associated with M, and δ_e is a delta function at identity $e \in G$.

Problem 1. Is it correct that the spectral measure μ associated with Markov operator on the Cayley graph of a finitely generated group determines it up to isometry?

μ determines the spectrum of M, probabilities $P_{e,e}^{(n)}$ of return, the Ihara zeta function, $\ldots$. Perhaps the answer could be affirmative.

The stated results are presented in [2].

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On inverse submonoids of the monoid of almost monotone injective co-finite partial selfmaps of positive integers

Oleg Gutik

We follow the terminology of [**1, 3, 4, 6, 7**].

The monoid $\mathscr{I}_{\infty}^{\Rsh\!\nearrow}(\mathbb{N})$ of almost monotone injective co-finite partial selfmaps of positive integers and the monoid $\mathscr{I}_{\infty}^{\nearrow}(\mathbb{N})$ of monotone injective co-finite partial selfmaps of positive integers studied in [**2**] and [**5**].

In our report we discuss on submonoids of the monoid $\mathscr{I}_{\infty}^{\Rsh\!\nearrow}(\mathbb{N})$. We give a small survey and announce new results on this topic.

Let $\mathscr{I}_{\infty}^{\nearrow}(\mathbb{N})$ be a submonoid of $\mathscr{I}_{\infty}^{\Rsh\!\nearrow}(\mathbb{N})$ which consists of cofinite monotone partial bijections of $\mathbb{N}$ and $\mathscr{C}_{\mathbb{N}}$ be a subsemigroup $\mathscr{I}_{\infty}^{\Rsh\!\nearrow}(\mathbb{N})$ which is generated by the partial shift $n \mapsto n+1$ and its inverse partial map. We show that every automorphism of a full inverse subsemigroup of $\mathscr{I}_{\infty}^{\nearrow}(\mathbb{N})$ which contains the semigroup $\mathscr{C}_{\mathbb{N}}$ is the identity map. We construct a submonoid $\mathbf{I}\mathbb{N}_{\infty}^{[\underline{1}]}$ of $\mathscr{I}_{\infty}^{\Rsh\!\nearrow}(\mathbb{N})$ with the following property: if S is an inverse submonoid of $\mathscr{I}_{\infty}^{\Rsh\!\nearrow}(\mathbb{N})$ such that S contains $\mathbf{I}\mathbb{N}_{\infty}^{[\underline{1}]}$ as a submonoid, then every non-identity congruence $\mathfrak{C}$ on S is a group congruence. We show that if S is an inverse submonoid of $\mathscr{I}_{\infty}^{\Rsh\!\nearrow}(\mathbb{N})$ such that S contains $\mathscr{C}_{\mathbb{N}}$ as a submonoid then S is simple and the quotient semigroup $S/\mathfrak{C}_{\mathbf{mg}}$, where $\mathfrak{C}_{\mathbf{mg}}$ is minimum group congruence on S, is isomorphic to the additive group of integers.

Also, we study topologizations of inverse submonoids of $\mathscr{I}_{\infty}^{\nearrow}(\mathbb{N})$ which contain $\mathscr{C}_{\mathbb{N}}$ and embeddings of such semigroups into compact-like topological semigroups.

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Hilbert polynomials for the algebra of invariants of binary d-form

Nadia Ilash

Let $\mathbb{K}$ be a field of characteristic zero. Let V_d be $d+1-$dimensional module of binary forms of degree d. Denote by $\mathbb{K}[V_d]^{SL_2}$ algebra of polynomial SL_2-invariant functions on V_d. In the language of classical invariant theory the algebra $\mathcal{I}_d := \mathbb{K}[V_d]^{SL_2}$ is called the algebra of invariants for binary form of degree d. It is well-known that $\mathcal{I}_d$ is finitely generated and graded:

$$\mathcal{I}_d = (\mathcal{I}_d)_0 \oplus (\mathcal{I}_d)_1 \oplus \ldots \oplus (\mathcal{I}_d)_n \oplus \ldots,$$

here $(\mathcal{I}_d)_n$ is a vector $\mathbb{K}-$space of invariants of degree n. Dimension of the vector space $(\mathcal{I}_d)_n$ is called *the Hilbert function* of the algebra $\mathcal{I}_d$. It is defined as a function of the variable n :

$$\mathcal{H}(\mathcal{I}_d, n) = \dim(\mathcal{I}_d)_n.$$

It is well-known that the Hilbert function of arbitrary finitely generated graded $\mathbb{K}$-algebra is quasi-polynomial (starting from some n), see **[1, 2, 3]**. Since the algebra of invariants $\mathcal{I}_d$ is finitely generated, we have

$$\mathcal{H}(\mathcal{I}_d, n) = h_0(n)n^r + h_1(n)n^{r-1} + ...,$$

where $h_k(n)$ is some periodic function with values in $\mathbb{Q}$. The quasi-polynomial $\mathcal{H}(\mathcal{I}_d, n)$ is called *the Hilbert polynomial* of algebra of invariants $\mathcal{I}_d$.

Let the contour $\mathbb{S}_1$ be the unit circle about 0. We obtain the following formula for computation of the Hilbert polynomials of the algebra $\mathcal{I}_d$:

$$\mathcal{H}(\mathcal{I}_d, n) = \frac{\cos^2 \frac{\pi n d}{2}}{2\pi i} \sum_{k=0}^{\frac{d-1}{2}} \oint_{\mathbb{S}_1} (-1)^{k+1} \frac{z^{\frac{k(k+1)}{2} - \left(\frac{d}{2} - k\right)n - 1}}{(z,z)_k (z^2, z)_{d-k-1}} dz,$$