then S is simple and the quotient semigroup $S/\mathfrak{C}_{\mathbf{mg}}$, where $\mathfrak{C}_{\mathbf{mg}}$ is minimum group congruence on S, is isomorphic to the additive group of integers.

Also, we study topologizations of inverse submonoids of $\mathscr{I}_{\infty}^{\nearrow}(\mathbb{N})$ which contain $\mathscr{C}_{\mathbb{N}}$ and embeddings of such semigroups into compact-like topological semigroups.

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Hilbert polynomials for the algebra of invariants of binary d-form

Nadia Ilash

Let $\mathbb{K}$ be a field of characteristic zero. Let V_d be $d+1-$dimensional module of binary forms of degree d. Denote by $\mathbb{K}[V_d]^{SL_2}$ algebra of polynomial SL_2-invariant functions on V_d. In the language of classical invariant theory the algebra $\mathcal{I}_d := \mathbb{K}[V_d]^{SL_2}$ is called the algebra of invariants for binary form of degree d. It is well-known that $\mathcal{I}_d$ is finitely generated and graded:

$$\mathcal{I}_d = (\mathcal{I}_d)_0 \oplus (\mathcal{I}_d)_1 \oplus \ldots \oplus (\mathcal{I}_d)_n \oplus \ldots,$$

here $(\mathcal{I}_d)_n$ is a vector $\mathbb{K}-$space of invariants of degree n. Dimension of the vector space $(\mathcal{I}_d)_n$ is called *the Hilbert function* of the algebra $\mathcal{I}_d$. It is defined as a function of the variable n :

$$\mathcal{H}(\mathcal{I}_d, n) = \dim(\mathcal{I}_d)_n.$$

It is well-known that the Hilbert function of arbitrary finitely generated graded $\mathbb{K}$-algebra is quasi-polynomial (starting from some n), see [**1, 2, 3**]. Since the algebra of invariants $\mathcal{I}_d$ is finitely generated, we have

$$\mathcal{H}(\mathcal{I}_d, n) = h_0(n)n^r + h_1(n)n^{r-1} + \ldots,$$

where $h_k(n)$ is some periodic function with values in $\mathbb{Q}$. The quasi-polynomial $\mathcal{H}(\mathcal{I}_d, n)$ is called *the Hilbert polynomial* of algebra of invariants $\mathcal{I}_d$.

Let the contour $\mathbb{S}_1$ be the unit circle about 0. We obtain the following formula for computation of the Hilbert polynomials of the algebra $\mathcal{I}_d$:

$$\mathcal{H}(\mathcal{I}_d, n) = \frac{\cos^2 \frac{\pi n d}{2}}{2\pi i} \sum_{k=0}^{\frac{d-1}{2}} \oint\limits_{\mathbb{S}_1} (-1)^{k+1} \frac{z^{\frac{k(k+1)}{2} - \left(\frac{d}{2}-k\right)n-1}}{(z,z)_k (z^2,z)_{d-k-1}} dz,$$

here $(a,q)_n = (1-a)(1-a\,q)\cdots(1-a\,q^{n-1})$ is $q-$shifted factorial.

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Some examples of even quandles and their automorphism groups

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Quandles are non-associative algebraic structures that are idempotent and distributive. The concept of quandles is still relatively new. Hence, this work is aimed at developping a new method of constructing quandles of finite even orders. Inner automorphism groups of the examples were obtained. The centralizer of certain elements of the quandles constructed were also obtained, and these were used to classify the constructed examples up to isomorphism.

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On semicommutative semigroups and abelian polygons

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We introduce the notions of semicommutative semigroups and abelian S-polygons by analogy with the notions of semicommutative, abelian modules and rings investigated in [**1**] and [**2**].

Definition 1. We say a semigroup S is a semicommutative semigroup if for any $x, y \in S$, $xy = 0$ implies $xSy = 0$.

Proposition 1. *For a semigroup S the following three statement are equivalent:*