On Bruck-Reilly λ-extensions of semigroups

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We follow the terminology of [1, 3, 5]. In 1970 Nivat and Perrot proposed the following generalization of the bicyclic monoid (see [4] and [3, Section 9.3]). For a non-zero cardinal λ, the polycyclic monoid on λ generators P_λ is the semigroup with zero given by the presentation:

$$P_\lambda = \left\langle \{p_i\}_{i\in\lambda}, \{p_i^{-1}\}_{i\in\lambda} \mid p_i p_i^{-1} = 1, p_i p_j^{-1} = 0 \text{ for } i \neq j \right\rangle.$$

By Remark 2 from [2] for every cardinal $\lambda \geqslant 2$ any non-zero element x of the polycyclic monoid P_λ has the form $u^{-1}v$, where u and v are elements of the free monoid $\mathscr{M}_\lambda$ over cardinal λ, and the semigroup operation on P_λ in this representation is defined in the following way:

$$a^{-1}b\cdot c^{-1}d = \begin{cases} (c_1a)^{-1}d, & \text{if } \ c = c_1 b \ \text{ for some } c_1 \in \mathscr{M}_\lambda;\\ a^{-1}b_1 d, & \text{if } \ b = b_1 c \ \text{ for some } b_1 \in \mathscr{M}_\lambda;\\ 0, & \text{otherwise,}\end{cases} \quad \text{and} \quad a^{-1}b\cdot 0 = 0\cdot a^{-1}b = 0\cdot 0 = 0.$$

Let S be a monoid and $\theta : S \to H_S(1)$ be a homomorphism into the group of units $H_S(1)$ of S. The set $(S \times (P_\lambda \setminus \{0\})) \sqcup \{0\}$ with the operation

$$(s, a_1^{-1}a_2)\cdot(t, b_1^{-1}b_2) = \begin{cases} (\theta^{|u|}(s)t, (ua_1)^{-1}b_2), & \text{if there exists } u \in \mathscr{M}_\lambda \text{ such that } b_1 = ua_2;\\ (s\theta^{|v|}(t), a_1^{-1}vb_2), & \text{if there exists } v \in \mathscr{M}_\lambda \text{ such that } a_2 = vb_1;\\ 0, & \text{otherwise,}\end{cases}$$

and $(s, a_1^{-1}a_2)\cdot 0 = 0\cdot(s, a_1^{-1}a_2) = 0\cdot 0 = 0$, where $\theta^n(s) = \underbrace{\theta\circ\ldots\circ\theta}_{n}(s)$ for any $n \in \mathbb{N}$ and $\theta^0(s) = s$ is called the *Bruck-Reilly λ-extension* of S with the homomorphism θ. and it will be denoted by $\mathscr{P}_\lambda(\theta, S)$.

In our report we discuss on algebraic property of $\mathscr{P}_\lambda(\theta, S)$ with the respect to the monoid S and the homomorphism θ. In particular we describe Green's relation on $\mathscr{P}_\lambda(\theta, S)$, preserving regularity, orthodoxity, inversability, combinatory, simplicity, ets, by the construction of the Bruck-Reilly λ-extension $\mathscr{P}_\lambda(\theta, S)$.

Also, we discuss on topologizations of the semigroup $\mathscr{P}_\lambda(\theta, S)$ and describe structure of some classes 0-bisimple semitopological semigroups which algebraic structure determines by so Bruck-Reilly λ-extensions of groups.

References

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