Anatoliy Petravchuk
Department of Mechanics and Mathematics, Taras Shevchenko National University of Kyiv, Kyiv, Ukraine
Email address: apetrav@gmail.com

Key words and phrases. Lie algebra, polynomial ring, derivation, rank of algebra, general affine Lie algebra

On eigenvalues of random partial wreath product

Eugenia Kochubinska, Olena Krytska

Partial wreath n-th power of symmetric inverse semigroup $\mathcal{I}_d$ is a semigroup defined recursively by

$$\mathcal{P}_n = (\mathcal{P}_{n-1}) \wr_p \mathcal{I}_d = \{(f, a) | a \in \mathcal{I}_d, f : \mathrm{dom}(a) \to \mathcal{P}_{n-1}\}, n \geq 2,$$

with composition

$$(f, a) \cdot (g, b) = (fg^a, ab),$$

and $\mathcal{P}_1 = \mathcal{I}_d$.

To a randomly chosen element $x \in \mathcal{P}_n$, we assign the matrix

$$A_x = \left(1_{\{x(v_i^n)=v_j^n\}}\right)_{i,j=1}^{d^n}.$$

In other words, (i, j)-th entry of A_x is equal to 1 if transformation x maps i to j, and 0 otherwise.

Let $\chi_x(\lambda)$ be the characteristic polynomial of A_x and $\lambda_1, \ldots, \lambda_{d^n}$ be its roots respecting multiplicity.

Denote by

$$\Xi_n = \frac{1}{d^n} \sum_{k=1}^{d^n} \delta_{\lambda_k}$$

a uniform distribution on eigenvalues of A_x.

Let $x \in \mathcal{P}_n$ let $\eta_n(x) = \Xi_n(0)$ be a fraction of zero eigenvalues A_x and $\xi_n(x) = 1 - \eta_n(x)$ be a fraction of nonzero eigenvalues. Then we have

$$\eta_n(x) \xrightarrow{\mathbb{P}} 1, n \to \infty,$$

or, equivalently,

$$\xi_n(x) \xrightarrow{\mathbb{P}} 0, n \to \infty.$$

Contact information

Eugenia Kochubinska
Department of Mechanics and Mathematics, Taras Shevchenko National University of Kyiv, Kyiv, Ukraine
Email address: eugenia@unv.kiev.ua

Olena Krytska
Department of Mechanics and Mathematics, Taras Shevchenko National University of Kyiv, Kyiv, Ukraine
Email address: lenuly95@gmail.com

Key words and phrases. Partial automorhism, wreath product, spectral measure

Classification of quasigroup functional equations and identities of minimal length

Halyna Krainichuk

A groupoid $(Q;\cdot)$ is called a *quasigroup*, if for all a, $b \in Q$ every of the equations $x \cdot a = b$ and $a \cdot y = b$ has a unique solution. A *σ-parastrophe* $(Q;\overset{\sigma}{\cdot})$ of $(Q;\cdot)$ is defined by

$$x_{1\sigma} \overset{\sigma}{\cdot} x_{2\sigma} = x_{3\sigma} \iff x_1 \cdot x_2 = x_3, \quad \sigma \in S_3.$$

A *σ-parastrophe of a class* of quasigroups $\mathfrak{A}$ is called a class ${}^{\sigma}\mathfrak{A}$, which consists of all σ-parastrophes of quasigroups from $\mathfrak{A}$ [**4**].

Two identities are called:

— *equivalent*, if they determine the same variety;
— *parastrophically equivalent*, if they determine parastrophic varieties.

Evidently that every equinelent identity are parastrophically equivalent, but the inverse is not valid.

A *parastrophic symmetry group* of a variety $\mathfrak{A}$ is $\mathrm{Ps}(\mathfrak{A}) := \{\sigma \mid {}^{\sigma}\mathfrak{A} = \mathfrak{A}\}$ and it is subgroup of the group S_3. A variety is called:

— totally-symmetric, if $\mathrm{Ps}(\mathfrak{A}) = S_3$;
— semisymmetric, if $\mathrm{Ps}(\mathfrak{A}) = A_3$;
— one-sided-symmetric, if $|\mathrm{Ps}(\mathfrak{A})| = 2$;
— asymmetric, if $|\mathrm{Ps}(\mathfrak{A})| = 1$.

A truss of varieties is called the set of all pairwise parastrophic varieties. A truss of varieties is uniquely defined by an identity which describes one of varieties from the given truss. A truss will be called: *totally-symmetric*, if it has 1 variety; *semisymmetric*, if it has two varieties; *one-sided-symmetric*, if it has three varieties; *asymmetric*, if it has six varieties.

A *length* of an identity is defined as the number of all functional symbols (not necessary different) appearing in it. Any quasigroup identity of length 1 is equivalent to the identity of idempotency.

Theorem 1. *An arbitrary quasigroup identity of length 2 is equivalent to exactly one of the following 14 identities and is parastrophically-equivalent to exactly one of the 6 identities having different numbers:*

$$\begin{array}{llll}
1)\ x = x, & 2)\ xy \cdot x = y, & & \\
3)\ xy = yx, & 4)\ x^2 = y^2, & 5)\ x^2 \cdot y = y, & 6)\ x^2 \cdot x = x, \\
{}^{\ell}3)\ x \cdot xy = y, & {}^{\ell}4)\ (x \overset{\ell}{\cdot} x)y = y, & {}^{s}5)\ x \cdot y^2 = x, & {}^{s}6)\ x \cdot x^2 = x, \\
{}^{r}3)\ xy \cdot y = x, & {}^{r}4)\ x(y \overset{r}{\cdot} y) = x, & {}^{\ell}5)\ x(y \overset{\ell}{\cdot} y) = x, & {}^{\ell}6)\ x(x \overset{\ell}{\cdot} x) = x.
\end{array}$$

Corollary 1. *All quasigroup identities of length 2 determine 14 different varieties distributing in 6 trusses according to the law of parastrophic symmetry. The trusses* 1), 2) *are totally-symmetric and the trusses* 3), 4), 5), 6) *are one-sided-symmetric.*