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A matrix representation of Fibonacci and Lucas polynomials

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The k-Fibonacci and k-Lucas polynomials [2] are the natural extension of the k-Fibonacci and k-Lucas numbers and many of their properties admit a straightforward proof. The Fibonacci sequence and the golden ratio have appeared in many fields of science including high energy physics, cryptography and coding [1, 5].

Definition 1. The Fibonacci polynomial $F_n(x)$ is defined recurrently relation

$$F_{n+1}(x) = xF_n(x) + F_{n-1}(x) \tag{1}$$

with $F_0(x) = 0$, $F_1(x) = 1$ for $n \geq 1$.

Fibonacci polynomials for negative subscripts are defined as $F_{-n}(x) = (-1)^{n+1}F_n(x)$ for $n \geq 1$.

Definition 2. The Lucas polynomial $L_n(x)$ is defined by the relation

$$L_{n+1}(x) = xL_n(x) + L_{n-1}(x) \tag{2}$$

with $L_0(x) = 2$, $L_1(x) = x$ for $n \geq 1$ and $L_n(x) = F_{n+1}(x) + F_{n-1}(x)$ for $n \in \mathbb{Z}$.

If $x = 1$, the classic Fibonacci and Lucas sequences are obtained from (1), (2) [3-5].

Lemma 1. *If X is a square matrix with $X^2 = xX + I$, then $X^n = F_n(x)X + F_{n-1}(x)I$ for all $n \in \mathbb{Z}$.*

Proposition 1. *Let* $Q(x) = \begin{pmatrix} x & 1 \\ 1 & 0 \end{pmatrix}$. *Then 1)* $Q(x)^n = \begin{pmatrix} F_{n+1}(x) & F_n(x) \\ F_n(x) & F_{n-1}(x) \end{pmatrix}$ *for all* $n \in \mathbb{Z}$; *2) det* $Q(x)^n = (-1)^n$ *(Cassini's identity).*

Proposition 2. *Let* $R(x) = \begin{pmatrix} x & 2 \\ 2 & -x \end{pmatrix}$. *Then 1)* $Q(x)R(x) = R(x)Q(x)$; *2)* $Q(x)^n R(x) = \begin{pmatrix} L_{n+1}(x) & L_n(x) \\ L_n(x) & L_{n-1}(x) \end{pmatrix}$ *for all* $n \in \mathbb{Z}$; *3) det* $(Q(x)^n R(x)) = (-1)^{n+1}(x^2+4)$ *(Cassini's identity).*

Proposition 3. *The n-th Fibonacci polynomial may be written as* $F_n(x) = \frac{\sigma^n - (-\sigma)^{-n}}{\sigma + \sigma^{-1}}$ *being* $\sigma = \frac{x+\sqrt{x^2+4}}{2}$ *(Binet's formula).*

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On the structure of finite groups whose non-normal subgroups are core-free

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Let G be a group. The following two normal subgroups are associated with any subgroup H of the group G: H^G, the normal closure of H in a group G, the least normal subgroup of G including H, and $\mathbf{Core}_G(H)$, the (normal) core of H in G, the greatest normal subgroup of G which is contained in H. We have $H^G = \langle H^x | x \in G\rangle$ and $\mathbf{Core}_G(H) = \bigcap_{x\in G} H^x$. A subgroup H is normal if and only if $H = H^G = \mathbf{Core}_G(H)$. In this sense, the subgroups H, for which $\mathbf{Core}_G(H) = \langle 1\rangle$, are the complete opposite of the normal subgroups. A subgroup H of a group G is called *core–free* in G if $\mathbf{Core}_G(H) = \langle 1\rangle$.

There is a whole series of papers devoted to the study of groups with only two types of subgroups. In particular, from the results of [**1**] it is possible to obtain a description of groups that have only two possibilities for each subgroup H: $H^G = H$ or $H^G = G$. In this connection, a dual question naturally arises on the structure of groups in which there are only two other possibilities: $\mathbf{Core}_G(H) = H$ or $\mathbf{Core}_G(H) = \langle 1\rangle$. The finite groups having this property have been studied in [**2**].