Proposition 1. *Let* $Q(x) = \begin{pmatrix} x & 1 \\ 1 & 0 \end{pmatrix}$. *Then 1)* $Q(x)^n = \begin{pmatrix} F_{n+1}(x) & F_n(x) \\ F_n(x) & F_{n-1}(x) \end{pmatrix}$ *for all* $n \in \mathbb{Z}$*; 2) det* $Q(x)^n = (-1)^n$ *(Cassini's identity).*

Proposition 2. *Let* $R(x) = \begin{pmatrix} x & 2 \\ 2 & -x \end{pmatrix}$. *Then 1)* $Q(x)R(x) = R(x)Q(x)$*; 2)* $Q(x)^n R(x) = \begin{pmatrix} L_{n+1}(x) & L_n(x) \\ L_n(x) & L_{n-1}(x) \end{pmatrix}$ *for all* $n \in \mathbb{Z}$*; 3) det* $(Q(x)^n R(x)) = (-1)^{n+1}(x^2+4)$ *(Cassini's identity).*

Proposition 3. *The n-th Fibonacci polynomial may be written as* $F_n(x) = \frac{\sigma^n - (-\sigma)^{-n}}{\sigma + \sigma^{-1}}$ *being* $\sigma = \frac{x+\sqrt{x^2+4}}{2}$ *(Binet's formula).*

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On the structure of finite groups whose non-normal subgroups are core-free

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Let G be a group. The following two normal subgroups are associated with any subgroup H of the group G: H^G, the normal closure of H in a group G, the least normal subgroup of G including H, and $\mathbf{Core}_G(H)$, the (normal) core of H in G, the greatest normal subgroup of G which is contained in H. We have $H^G = \langle H^x | x \in G\rangle$ and $\mathbf{Core}_G(H) = \bigcap_{x\in G} H^x$. A subgroup H is normal if and only if $H = H^G = \mathbf{Core}_G(H)$. In this sense, the subgroups H, for which $\mathbf{Core}_G(H) = \langle 1\rangle$, are the complete opposite of the normal subgroups. A subgroup H of a group G is called *core–free* in G if $\mathbf{Core}_G(H) = \langle 1\rangle$.

There is a whole series of papers devoted to the study of groups with only two types of subgroups. In particular, from the results of [**1**] it is possible to obtain a description of groups that have only two possibilities for each subgroup H: $H^G = H$ or $H^G = G$. In this connection, a dual question naturally arises on the structure of groups in which there are only two other possibilities: $\mathbf{Core}_G(H) = H$ or $\mathbf{Core}_G(H) = \langle 1\rangle$. The finite groups having this property have been studied in [**2**].

Our main result gives a description of the finite soluble groups, whose non–normal subgroups are core–free. As we noted above, the finite groups whose non–normal subgroups are core–free were studied in [**2**]. Our description is more detailed than the description given in Theorem 1 of this paper. We also note that the proof of Lemma 5 of the paper [**2**] contains a gap (only the case when the both factor–groups G/N_1 and G/N_2 are non–abelian) and mistake (the fact that H is a subgroup of $T \times A$ does not imply that $H = H_1 \times H_2$ where $H_1 \leqslant T$ and $H_2 \leqslant A$).

Let G be a group. The intersection of all non–trivial normal subgroups $\mathbf{Mon}(G)$ of G is called the *monolith* of a group G. If $\mathbf{Mon}(G) \neq \langle 1 \rangle$, then the group G is called *monolithic*.

Theorem 1. *Let G be a finite soluble group, whose non–normal subgroups are core–free. Suppose that G is not a Dedekind group. Then G is monolithic.*

If the center of G includes the monolith, then $G = KE$ where K is a cyclic p–subgroup, E is an extraspecial p–subgroup, $K = \zeta(G)$ and $K \cap E = [G, G]$ is a subgroup of order p, p is a prime.

If the monolith of G is not central, then $G = \mathbf{Mon}(G) \leftthreetimes A$, and:

(i) $\mathbf{Mon}(G)$ *is elementary abelian p–subgroup for some prime p and A is a p'-group;*
(ii) $[G, G] = \mathbf{Mon}(G) = C_G(\mathbf{Mon}(G))$*;*
(iii) *whether the subgroup A is cyclic, or $A = Q \times B$ where Q is a quaternion group of order* 8*, and B is a cyclic $2'$–subgroup;*
(iv) *if C is another complement to* $\mathbf{Mon}(G)$ *in G, then the subgroups A and C are conjugate.*

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