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Separability of the lattice of τ-closed totally ω-composition formations of finite groups

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All groups considered are finite. All notations and terminologies are standard [**1**]-[**5**].

Let ω be a non-empty set of primes. Every function of the form $f : \omega \bigcup \{\omega'\} \to \{\text{formations}\}$ is called an *ω-composition satellite*. For any ω-composition satellite $CF_\omega(f) = \{G | G/R_\omega(G) \in f(\omega')$ and $G/C^p(G) \in f(p)$ for all $p \in \pi(Com(G)) \cap \omega\}$. If the formation $\mathfrak{F}$ is such that $\mathfrak{F} = CF_\omega(f)$ for some ω-composition satellite f, then it is ω-composition formation, and f — ω-composition satellite of this formation.

Every formation of groups is called *0-multiply ω-composition*. For $n \geq 1$, a formation $\mathfrak{F}$ is called *n-multiply ω-composition*, if it has an ω-composition satellite f such that every value $f(p)$ of f is an $(n-1)$-multiply ω-composition formation. A formation $\mathfrak{F}$ is called *totally ω-composition* if it is n-multiply ω-composition for all natural n.

Let for any group G, $\tau(G)$ be a set of subgroups of G such that $G \in \tau(G)$. Then we say following [**5**] that τ is a *subgroup functor* if for every epimorphism $\varphi : A \to B$ and any groups $H \in \tau(A)$ and $T \in \tau(B)$ we have $H^\varphi \in \tau(B)$ and $T^{\varphi^{-1}} \in \tau(A)$. A class $\mathfrak{F}$ of groups is called *τ-closed* if $\tau(G) \subseteq \mathfrak{F}$ for all $G \in \mathfrak{F}$.

Let $\mathfrak{X}$ be some set of groups. Then $c^\tau_{\omega_\infty}$form $\mathfrak{X}$ denotes the totally ω-composition formation generated by $\mathfrak{X}$, i.e. $c^\tau_{\omega_\infty}$form$\mathfrak{X}$ is the intersection of all τ-closed totally ω-composition formations containing $\mathfrak{X}$. For any two τ-closed totally ω-composition formations $\mathfrak{M}$ and $\mathfrak{H}$, we write $\mathfrak{M} \vee^\tau_{\omega_\infty} \mathfrak{H} = c^\tau_{\omega_\infty}\text{form}(\mathfrak{M} \cup \mathfrak{H})$.

With respect to the operations $\vee^\tau_{\omega_\infty}$ and $\cap$ the set $c^\tau_{\omega_\infty}$ of all τ-closed totally ω-composition formations forms a complete lattice. Formations in $c^\tau_{\omega_\infty}$ are called *$c^\tau_{\omega_\infty}$-formations.*

Let $\mathfrak{X}$ be a non-empty class of finite groups. A complete lattice θ of formations is called $\mathfrak{X}$-separable, if for every term $\nu(x_1, ..., x_n)$ of signature $\{\cap, \vee_\theta\}$, θ-formations $\mathfrak{F}_1$, ..., $\mathfrak{F}_n$ and every group $A \in \mathfrak{X} \cap \nu(\mathfrak{F}_1, ..., \mathfrak{F}_n)$ are exists $\mathfrak{X}$-groups $A_1 \in \mathfrak{F}_1, ..., A_n \in \mathfrak{F}_n$ such that $A \in \nu(\theta\text{form}A_1, ..., \theta\text{form}A_n)$. In particular, if $\mathfrak{X} = \mathfrak{G}$ is the class of all finite groups then the lattice θ of formations is called $\mathfrak{G}$-separable or separable.

Theorem 1. *The lattice $c^\tau_{\omega_\infty}$ all τ-closed totally ω-composition formations is $\mathfrak{G}$-separated.*

Let τ be the trivial subgroup functor or let ω be the set of all primes. Then we obtain

Corollary 1. *The lattice c_∞^ω all totally ω-composition formations is $\mathfrak{G}$-separated.*

Corollary 2. *The lattice c_∞^τ all τ-closed totally composition formations is $\mathfrak{G}$-separated.*

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Definition of invertibility property for loops via translations

Alla Lutsenko

A *quasigroup* can be defined as a groupoid $(Q;\cdot)$ in which all *left translations* L_a ($L_a(x) := a\cdot x$) and all *right translations* R_a ($R_a(x) := x\cdot a$) are bijections of the carrier Q. In a quasigroup, a definition of a *middle translation* M_a ($M_a(x) = y :\Leftrightarrow xy = a$) is also possible. Therefore, an element e of a quasigroup is *neutral*, if left and right translations defined by e are identical transformations of the carrier: $L_e = R_e = \iota$. A quasigroup having a neutral element is called a *loop*.

The invertibility property also can be defined via translations of a quasigroup. Rememder that a quasigroup has [**1, 2**]:

- a *left inverse property* (briefly, a *left IP-quasigroup*), if there is a transformation λ such that for all x, y $\lambda x\cdot xy = y$;
- a *right inverse property* (briefly, a *right IP-quasigroup*), if there is a transformation ρ such that for all x, y $yx\cdot\rho x = y$;
- a *left cross inverse property* (briefly, a *left CIP-quasigroup*), if there is a transformation γ such that for all x, y $\gamma(x)\cdot yx = y$;
- a *right cross inverse property* (briefly, a *right CIP-quasigroup*), if there is a transformation δ such that for all x, y $xy\cdot\delta(x) = y$.

The defining equalities can be written as $L_{\lambda x}L_x = \iota$, $R_{\rho x}R_x = \iota$, $L_{\gamma x}R_x = \iota$, $R_{\delta x}L_x = \iota$ respectively [**1**], i.e.,

$$L_x^{-1} = L_{\lambda x}, \qquad R_x^{-1} = R_{\rho x}, \qquad R_x^{-1} = L_{\gamma x}, \qquad L_x^{-1} = R_{\delta x}.$$

Thus, the common property for all these classes of quasigroups is the following: ***each translation of a quasigroup is also a translation of the quasigroup.***