On non-periodic groups with non-Dedekind locally nilpotent norm of decomposable subgroups

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In group theory the findings related to the study of groups, subgroups (or the system of subgroups) of which have some theoretical group property, have given restrictions, are in the focus. In some cases the presence of one characteristic subgroup with a certain property can be the determining factor for the structure of the group. Different Σ-norms of a group are the subgroups of such a type.

The authors continue the study of different classes of groups with non-Dedekind norm of decomposable subgroups, started in [**1**]–[**2**]. *Decomposable subgroup* is a subgroup of a group G representable in the form of the direct product of two nontrivial factors [**3**]. The intersection N_G^d of normalizers of all decomposable subgroups of the group G is called *the norm of decomposable subgroups*. If $N_G^d = G$, then either all decomposable subgroups are normal in G or the set of such subgroups are empty. Non-Abelian groups with such a property were studied in [**3**] and called *di-groups*. The characterization of non-periodic locally solvable groups with non-Dedekind locally nilpotent norm N_G^d of decomposable subgroups are given in the article.

Theorem 1. *Non-periodic locally solvable and not locally nilpotent group G has non-Dedekind locally nilpotent norm N_G^d of decomposable subgroups if and only if G is a group of one of the following types:*

1) *$G = B\lambda\langle y, q\rangle$, where $|y| = 8$, $y^4 = q^2$, $q^{-1}yq = y^{-1}$, B is an Abelian torsion free group of rank 1, $y^{-1}by = b^{-1}$, $[q, b] = 1$ for any element $b \in B$; $N_G^d = \langle y^2, q\rangle \times B$;*
2) *$G = \langle a\rangle \lambda B\langle y\rangle$, $|a| = p^n$, p is a prime, $p \neq 2$, $n > 1$, $B\langle y\rangle$ is an Abelian torsion free group of rank 1, B is a not p-divisible Abelian torsion free group of rank 1, $|y| = \infty$, $[B\langle y\rangle : B] = k$, $k \in \mathbb{N}$, $k|(p-1)$, $k > 1$, $[\langle a\rangle, \langle y\rangle] = \langle a\rangle$, $[\langle a\rangle, B] = \left\langle a^{p^{n-1}}\right\rangle$; $N_G^d = \langle a\rangle \lambda B$;*
3) *$G = (\langle a\rangle \lambda B)\langle g\rangle$, $|a| = 4$, B is a not 2-divisible Abelian torsion free group of rank 1, $[\langle a\rangle, B] = \langle a^2\rangle$, $g^2 = a$. If $b \in B\backslash C_G(a)$, then $g^{-1}bg = b^{-1}a^{-1}$. If $b \in B \cap C_G(a)$, then $g^{-1}bg = b^{-1}$; $N_G^d = \langle a\rangle \lambda B$.*

References

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The Structure Properties of Rational Numbers Are Important Component of Mathematical Knowledge of Mathematics Teachers

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An outstanding German mathematician and educator F. Klein (1849 – 1925) devoted much effort to substantiate the ideas of the modernization of school mathematical education. In [**1**], he critically evaluated the situation with the teaching of mathematics at school. According to F. Klein and the successor to his ideas Dutch mathematician and popularizer of this science G. Freudenthal (1905 – 1990) it is absolutely unacceptable for a school to remain alien to all that constitutes the content of modern mathematics [**2**], [**3**]. Now a step forward in the teaching of elementary mathematics would be to study the problems of elementary mathematics from the standpoint of modern higher mathematics, in which are dominated by structural approaches.

There are investigated some structure properties of field $(\mathbb{Q};+,\cdot;0,1)$ rational numbers, some properties of its subfields, some properties of subgroups of additive group $(\mathbb{Q};+;0)$ and multiplicative group $(\mathbb{Q}\setminus 0;;1)$ of this field in this talk.

One of the basic subrings of rational numbers field is integer numbers ring. The stimulus to its extension to minimal numeral field (which are rational numbers field) is the problem of equation's $ax=b$ with integer coefficients soluble. When such equation has a solution with $a\neq 0$, the minimal field condition gives an answer about representation any rational number as a quotient of two integer numbers.

Thus, the rational numbers set $\mathbb{Q}=\mathbb{Z}\cup\mathbb{Q}\setminus\mathbb{Z}$, when $\mathbb{Z}$ denotes the integer numbers set and $\mathbb{Q}\setminus\mathbb{Z}$ denotes the fraction numbers set. The uniquely representation any rational number $q\neq 0$ as a two integer numbers quotient is commonly known. But uniquely representations any rational number exist infinitely a lot. For example, next uniquely representation any rational number is interesting and useful for many problems. This representation is if $q>0$ then $q=p^n\frac{a}{b}$ when p — prime number, $n\in\mathbb{Z}$, a and b are natural numbers being $(a,b)=(a,p)=(b,p)=1$; if $q<0$ then $q=-p^n\frac{a}{b}$.

On subject of rings of rational numbers field they are consider the issues about their discreteness and density. It's proved, in particular, that every some ring of rational numbers field is density when fractional number belongs to it.

When we investigated the properties of numeral fields which rational numbers field don't have, it's showed the incompleteness of this field. It's proved this fact without using the irrational numbers.

It's suggested the one of possible proof that the group of automorphisms of group $(\mathbb{Q};+;0)$ is isomorphic to group $(\mathbb{Q}\setminus 0;\cdot;1)$, when we consider the additive and multiplicative groups of rational numbers field. It's proved that the group of automorphisms of group's $(\mathbb{Q};+;0)$ subgroups is isomorphic to subgroups of group $(\mathbb{Q}\setminus 0;\cdot;1)$ too. The last fact is illustrated by an example of group $(\mathbb{Z};+;0)$ integer numbers and an example of group $(\mathbb{Q}_p;+;0)$ p-adic numbers for any prime number p.