Marina Drushlyak
Department of Mathematics, Makarenko Sumy State Pedagogical University, Sumy, Ukraine
Email address: marydru@fizmatsspu.sumy.ua

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The Structure Properties of Rational Numbers Are Important Component of Mathematical Knowledge of Mathematics Teachers

Fedir Lyman, Oksana Odintsova

An outstanding German mathematician and educator F. Klein (1849 – 1925) devoted much effort to substantiate the ideas of the modernization of school mathematical education. In [**1**], he critically evaluated the situation with the teaching of mathematics at school. According to F. Klein and the successor to his ideas Dutch mathematician and popularizer of this science G. Freudenthal (1905 – 1990) it is absolutely unacceptable for a school to remain alien to all that constitutes the content of modern mathematics [**2**], [**3**]. Now a step forward in the teaching of elementary mathematics would be to study the problems of elementary mathematics from the standpoint of modern higher mathematics, in which are dominated by structural approaches.

There are investigated some structure properties of field $(\mathbb{Q}; +, \cdot; 0, 1)$ rational numbers, some properties of its subfields, some properties of subgroups of additive group $(\mathbb{Q}; +; 0)$ and multiplicative group $(\mathbb{Q} \setminus 0; ; 1)$ of this field in this talk.

One of the basic subrings of rational numbers field is integer numbers ring. The stimulus to its extension to minimal numeral field (which are rational numbers field) is the problem of equation's $ax = b$ with integer coefficients soluble. When such equation has a solution with $a \neq 0$, the minimal field condition gives an answer about representation any rational number as a quotient of two integer numbers.

Thus, the rational numbers set $\mathbb{Q} = \mathbb{Z} \cup \mathbb{Q} \setminus \mathbb{Z}$, when $\mathbb{Z}$ denotes the integer numbers set and $\mathbb{Q} \setminus \mathbb{Z}$ denotes the fraction numbers set. The uniquely representation any rational number $q \neq 0$ as a two integer numbers quotient is commonly known. But uniquely representations any rational number exist infinitely a lot. For example, next uniquely representation any rational number is interesting and useful for many problems. This representation is if $q > 0$ then $q = p^n \frac{a}{b}$ when p — prime number, $n \in \mathbb{Z}$, a and b are natural numbers being $(a, b) = (a, p) = (b, p) = 1$; if $q < 0$ then $q = -p^n \frac{a}{b}$.

On subject of rings of rational numbers field they are consider the issues about their discreteness and density. It's proved, in particular, that every some ring of rational numbers field is density when fractional number belongs to it.

When we investigated the properties of numeral fields which rational numbers field don't have, it's showed the incompleteness of this field. It's proved this fact without using the irrational numbers.

It's suggested the one of possible proof that the group of automorphisms of group $(\mathbb{Q}; +; 0)$ is isomorphic to group $(\mathbb{Q} \setminus 0; \cdot; 1)$, when we consider the additive and multiplicative groups of rational numbers field. It's proved that the group of automorphisms of group's $(\mathbb{Q}; +; 0)$ subgroups is isomorphic to subgroups of group $(\mathbb{Q} \setminus 0; \cdot; 1)$ too. The last fact is illustrated by an example of group $(\mathbb{Z}; +; 0)$ integer numbers and an example of group $(\mathbb{Q}_p; +; 0)$ p-adic numbers for any prime number p.

The teachers of Mathematics may make the knowledge of their students more deepen and more modern with all these facts.

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Contact information

Fedir Lyman
Department of Mathematics, Anton Makarenko Sumy State Pedagogical University, Sumy, Ukraine
Email address: mathematicsspu@gmail.com

Oksana Odintsova

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Systems of encoding of real numbers with infinite alphabet and mathematical objects with fractal properties

Iryna Lysenko, Mykola Pratsiovytyi

The expansions of fractional part of real number in positive series of Engel, Lüroth, Sylvester and alternating series of Ostrogradsky–Sierpiński–Pierce, Lüroth, Ostrogradsky are classic systems of encoding (representation) of numbers by means of infinite alphabet. Generalizations and analogues of such expansions are Q_∞- and $\widetilde{Q}_\infty$-expansion, continued fraction expansion, etc. These systems are used effectively in theory of fractals, metric number theory, probabilistic number theory, constructive theory of functions with fractal properties, geometry of numerical series, theory of singular probability distributions supported on fractals, etc.

The objects with a special topological and metric structure and fractal properties arise in the study of transformations and mappings. Transformations preserving fractal dimension (like a Hausdorff–Besicovitch dimension) of all Borel subsets of unit interval as well as mappings preserving tails of representation and frequency of digits form a group with respect to the operation of composition (superposition).

The talk is devoted to a pair of dual representations of real numbers with infinite alphabet, namely, positive Sylvester series and alternating Ostrogradsky series (in usual and difference form). We discuss the problem of representation of the same number in these systems of encoding. Properties of (left and right) shift operators are described. Transformations preserving tails of both representations are also constructed. We apply these representations to develop theory of probability distributions with fractal properties, constructive theory of singular and nowhere monotonic continuous functions, and nonmonotonic functions of Cantor type.

Recall that, for any number $x \in (0,1)$, there exist sequences of positive integers (g_n), (q_n) such that

$$x = \frac{1}{g_1} + \frac{1}{g_2} + ... + \frac{1}{g_n} + ... \equiv \Delta^S_{g_1 g_2 ... g_n ...}, 2 \leq g_1 \in N, g_{n+1} \geq q_n(g_n - 1) + 1, \tag{1}$$