The teachers of Mathematics may make the knowledge of their students more deepen and more modern with all these facts.

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Systems of encoding of real numbers with infinite alphabet and mathematical objects with fractal properties

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The expansions of fractional part of real number in positive series of Engel, Lüroth, Sylvester and alternating series of Ostrogradsky–Sierpiński–Pierce, Lüroth, Ostrogradsky are classic systems of encoding (representation) of numbers by means of infinite alphabet. Generalizations and analogues of such expansions are Q_∞- and $\widetilde{Q}_\infty$-expansion, continued fraction expansion, etc. These systems are used effectively in theory of fractals, metric number theory, probabilistic number theory, constructive theory of functions with fractal properties, geometry of numerical series, theory of singular probability distributions supported on fractals, etc.

The objects with a special topological and metric structure and fractal properties arise in the study of transformations and mappings. Transformations preserving fractal dimension (like a Hausdorff–Besicovitch dimension) of all Borel subsets of unit interval as well as mappings preserving tails of representation and frequency of digits form a group with respect to the operation of composition (superposition).

The talk is devoted to a pair of dual representations of real numbers with infinite alphabet, namely, positive Sylvester series and alternating Ostrogradsky series (in usual and difference form). We discuss the problem of representation of the same number in these systems of encoding. Properties of (left and right) shift operators are described. Transformations preserving tails of both representations are also constructed. We apply these representations to develop theory of probability distributions with fractal properties, constructive theory of singular and nowhere monotonic continuous functions, and nonmonotonic functions of Cantor type.

Recall that, for any number $x \in (0,1)$, there exist sequences of positive integers (g_n), (q_n) such that

$$x = \frac{1}{g_1} + \frac{1}{g_2} + ... + \frac{1}{g_n} + ... \equiv \Delta^S_{g_1 g_2 ... g_n ...}, 2 \leq g_1 \in N, g_{n+1} \geq q_n(g_n - 1) + 1, \quad (1)$$

$$x = \Delta^S_{g_1 g_2 \ldots g_n \ldots} = \overline{\Delta}^s_{a_1 a_2 \ldots a_n, \ldots}, a_1 = g_1, a_{n+1} = g_{n+1} - g_n(g_n - 1); \tag{2}$$

$$x = \frac{1}{q_1} - \frac{1}{q_2} + \ldots + (-1)^{n-1}\frac{1}{q_n} + \ldots \equiv \Delta^{O_2}_{q_1 q_2 \ldots q_n \ldots}, q_n \in N, q_{n+1} \geq q_n(q_n + 1), \tag{3}$$

$$x = \Delta^{O_2}_{q_1 q_2 \ldots q_n} = \Delta^{\overline{O}_2}_{d_1 d_2 \ldots d_n, \ldots}, d_1 = q_1, d_n = q_n + 1 - q_{n-1}(q_{n-1} + 1). \tag{4}$$

Expression (1) is called the expansion of number x by Sylvester series and notation $\Delta^S_{g_1 g_2 \ldots g_n \ldots}$ is called the representation of number by Sylvester series. After conversion of the representation by means of the same alphabet we get the difference form of representation by Sylvester series (2).

Similarly, expression (3) is called the expansion of number by Ostrogradsky series. Corresponding representation (4) obtained by conversion is called the difference form of representation by Ostrogradsky series.

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Filtered cocategories

Volodymyr Lyubashenko

Conilpotent cocategories are interesting thanks to the main example, tensor cocategories, which are related to A_∞-categories. A_∞-categories arising in symplectic geometry (Fukaya categories) are naturally filtered. We consider filtered cocategories foreseeing the applications to filtered A_∞-categories. A filtration on rings, modules, quivers, cocategories gives rise to a uniform structure, hence, to completion. We shall concentrate on complete cocategories, especially, on completed tensor cocategories. The main result claims that coderivations form a kind of internal hom for the monoidal category of completed tensor cocategories. As a corollary we get a multiplication on tensor cocategories of coderivations.

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