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Homological algebra in degree zero

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The term "homological algebra in degree zero" refers, in the narrow sense of the word, to calculation of the zeroth derived functor of an additive functor between abelian categories. Most people do not realize that this is an interesting problem because the zeroth right derived functor of a Hom functor is the same Hom functor, and the same can be said about the zeroth left derived functor of the tensor product. The situation changes dramatically if those functors are derived on the opposite sides. In fact, the emerging phenomena seem to be rather diverse and widespread. Those include: a new approach to classical torsion and to Bass torsion, a definition of cotorsion (this is a new concept), dualities between torsion and cotorsion, theorems of Watts and Eilenberg, new results in ring and module theory, a number of formulas that extend several of Auslander's formulas from finitely presented modules to arbitrary modules, a new generalization of Tate homology, a connection between module theory and stable homotopy theory, etc. In this talk, I will try and explain some of those results. Most of this talk is based on joint work with Jeremy Russell.

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Some subsemimodules of differential semimodules satisfying the ascending chain condition

Ivanna Melnyk

Let R be a semiring and let M be a left semimodule over R. A map $\delta\colon R \to R$ is called a *derivation on* R [**2**] if $\delta(a+b) = \delta(a) + \delta(b)$ and $\delta(ab) = \delta(a)\, b + a\delta(b)$ for any $a, b \in R$. A semiring R equipped with a derivation δ is called a *differential semiring* with respect to the derivation δ [**1**]. A map $d\colon M \to M$ is called a *derivation* of the semimodule M, associated with the semiring derivation $\delta\colon R \to R$ if $d(m+n) = d(m) + d(n)$ and $d(rm) = \delta(r)\, m + rd(m)$ for any $m, n \in M$, $r \in R$. A left R-semimodule M together with a derivation $d\colon M \to M$ is called a *differential semimodule.*

A subsemimodule N of the differential R-semimodule M is called *differential* if $d(N) \subseteq N$. For a subset X of M its *differential* $X_{\#}$ is defined to be the set $X_{\#} = \{x \in M \,|\, d^n(x) \in X \text{ for all } n \in \mathbb{N}_0\}$.

Let S be an m-system of R. A non-empty subset T of the R-semimodule M is called an *Sm-system* of M if for every $s \in S$ and $x \in T$ there exists $r \in R$ such that $srx \in T$.

A differential subsemimodule N of the differential semimodule M is called *quasi-prime* if it is maximal among differential subsemimodules of M disjoint from some Sm-system of M.

Theorem 1. *For a differential subsemimodule Q of R the following conditions are equivalent:*

(1) Q is a quasi-prime subsemimodule of M;
(2) $rad(Q)$ is a prime subsemimodule of M and $Q = (rad(Q))_{\#}$;
(3) There exists a prime subsemimodule P of M such that $Q = P_{\#}$.

Theorem 2. *For every differential subsemimodule N of the differential semimodule M satisfying the ascending chain condition on differential k-subsemimodules the following conditions are equivalent:*

(1) N is a differentially prime subsemimodule;
(2) N is a quasi-prime subsemimodule;
(3) $N = P_{\#}$ for some prime subsemimodule P of M.

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