3. A. Nowicki, *Polynomial Derivations and their Rings of Constants*, Uniwersytet Mikolaja Kopernika, Torun, 1994.

Contact information

Anatoliy Petravchuk
Department of Mechanics and Mathematics, Taras Shevchenko National University of Kyiv, Kyiv, Ukraine
Email address: apetrav@gmail.com

Kateryna Sysak
Department of Information Systems and Technologies, National Transport University, Kyiv, Ukraine
Email address: sysakkya@gmail.com

Key words and phrases. Lie algebra, polynomial ring, derivation, rank of algebra, Darboux polynomial

Classification of quasigroups according to their parastrophic symmetry groups

Yevhen Pirus

Let Q be a set, a mapping $f : Q^3 \to Q$ is called an invertible ternary operation (=function), if it is invertible element in all semigroups $(\mathcal{O}_3; \underset{0}{\oplus})$, $(\mathcal{O}_3; \underset{1}{\oplus})$ and $(\mathcal{O}_3; \underset{2}{\oplus})$, where $\mathcal{O}_3$ is the set of all ternary operations defined on Q and

$$(f \underset{1}{\oplus} f_1)(x_1, x_2, x_3) := f(f_1(x_1, x_2, x_3), x_2, x_3), \quad (f \underset{2}{\oplus} f_1)(x_1, x_2, x_3) := f(x_1, f_1(x_1, x_2, x_3), x_3),$$

$$(f \underset{3}{\oplus} f_1)(x_1, x_2, x_3) := f(x_1, x_2, f_1(x_1, x_2, x_3)).$$

The set of all ternary invertible functions is denoted by Δ_3. If an operation f is invertible and ${}^{(14)}f$, ${}^{(24)}f$, ${}^{(34)}f$ are its inverses in those semigroups, then the algebra $(Q; f, {}^{(14)}f, {}^{(24)}f, {}^{(34)}f)$ (in brief, $(Q; f)$) is called a *ternary quasigroup* [**1**]. The inverses are also invertible. All inverses to inverses are called *σ-parastrophes* of the operation f and can be defined by

$${}^{\sigma}f(x_{1\sigma}, x_{2\sigma}, x_{3\sigma}) = x_{4\sigma} \quad :\Leftrightarrow \quad f(x_1, x_2, x_3) = x_4, \quad \sigma \in S_4,$$

where S_4 denotes the group of all bijections of the set $\{0, 1, 2, 3\}$. Therefore in general, every invertible operation has 24 parastrophes. Since parastrophes of a quasigroup satisfy the equalities ${}^{\sigma}({}^{\tau}f) = {}^{\sigma\tau}f$, then the symmetric group S_4 defines an action on the set Δ_3. In particular, the fact implies that the number of different parastrophes of an invertible operation is a factor of 24. More precisely, it is equal to $24/|\mathrm{Ps}(f)|$, where $\mathrm{Ps}(f)$ denotes a stabilizer group of f under this action which is called *parastrophic symmetry group* of the operation f.

Let $\mathfrak{P}(H)$ denote the class of all quasigroups whose parastrophic symmetry group contains the group $H \in S_4$. A ternary quasigroup $(Q; f)$ belongs to $\mathfrak{P}(H)$ if and only if ${}^{\sigma}f = f$ for all σ from a set G of generators of the group H, therefore, the class of quasigroup $\mathfrak{P}(H)$ is a variety.

For every subgroup H of the group S_4 the variety $\mathfrak{P}(H)$ are described and its subvariety of ternary group isotopes are found. For example, let

$$D_8 := \{\iota, (12), (34), (12)(34), (13)(24), (14)(23), (1324), (1423)\} \le S_4.$$

Theorem 1. *A ternary quasigroup $(Q; f)$ belong to the variety $\mathfrak{P}(D_8)$ if and only if*

$$f(x, y, z) = f(y, x, z), \quad f(x, y, f(x, y, z)) = z, \quad f(z, f(x, y, z), x) = y. \tag{1}$$

$(Q; f)$ is called a *group isotope*, if there is a group $(G; \cdot)$ and bijections α, β, γ, δ such that

$$f(x, y, z) = \delta^{-1}(\alpha x \cdot \beta y \cdot \gamma z).$$

THEOREM 2. *A ternary group isotope (Q, f) belongs to $\mathfrak{P}(D_8)$ iff there exists an abelian group $(Q, +, 0)$, its involutive automorphism α and an element $a \in Q$ such that $\alpha a = -a$ and*

$$f(x_1, x_2, x_3) = \alpha x_1 + \alpha x_2 - x_3 + a.$$

References

1. Sokhatsky F. M. *Parastrophic symmetry in quasigroup theory* // Visnyk Donetsk national university, Ser. : natural sciences. — 2016. — No. 1–2. — P. 70–83.
2. Krainichuk H. *Classification of group isotopes according to their symmetry groups* // Folia Math. — 2017. — Vol. 19, no. 1. — P. 84–98.

CONTACT INFORMATION

Yevhen Pirus
Donetsk Regional Center for Educational Quality Assessment, Ukraine
Email address: pirus@ukr.net

Key words and phrases. Functional equations, quasigroups, group isotopes, mediality, invertible functions

Cohomology of lattices over finite abelian groups

ANDRIANA PLAKOSH

It is a joint work with Yuriy Drozd.

Let G be a finite abelian group which is a direct product of cyclic groups $C_1 \times C_2 \times \cdots \times C_s$, where $\#(C_k) = o_k$. We propose a new resolution for the trivial G-module $\mathbb{Z}$ that simplifies the calculations of cohomologies. We also study the cohomology of G-*lattices*, i.e. G-module M such that the abelian group of M is free of finite rank. We set $M^* = \mathrm{Hom}_{\mathbb{Z}}$ and $DM = \mathrm{Hom}_G(M, \mathbb{T})$, where $\mathbb{T} = \mathbb{Q}/\mathbb{Z}$. We prove the following results about Tate cohomologies $\hat{H}^n(G, M)$, where M is a G-lattice. We establish a duality theorem generalizing [**1**, Theorem XII.6.6].

THEOREM 1.

$$\hat{H}^{n-1}(G, DM) \simeq D\hat{H}^{-n}(G, M),$$
$$\hat{H}^{n}(G, DM) \simeq \hat{H}^{n+1}(G, M^*),$$
$$\hat{H}^{n}(G, M^*) \simeq D\hat{H}^{-n}(G, M).$$

As $\hat{H}^n(G, M)_p \simeq \hat{H}^n(G_p, M)$, where G_p is the p-part of G, we suppose further that G is a p-group and $o_k = p^{m_k}$, where $m_1 \geq m_2 \geq \cdots \geq m_s$. Recall that a G-lattice M is called *irreducible* if so is the $\mathbb{Q}G$-module $\mathbb{Q} \otimes_{\mathbb{Z}} M$.

Set $\nu(n, s) = (-1)^n \sum_{i=0}^{n} \binom{-s}{i}$.

THEOREM 2. *Let M be an irreducible G-lattice*

(1) If $M \not\simeq \mathbb{Z}$, then $\hat{H}^n(G, M) \simeq (\mathbb{Z}/p\mathbb{Z})^{\nu(|n|-1, s)}$.

(2) If $n \neq 0$, then $\hat{H}^n(G, \mathbb{Z}) \simeq \bigoplus_{k=1}^{s} (\mathbb{Z}/p^{m_k}\mathbb{Z})^{\nu(|n|-1, k)+(-1)^n}$.

Recall that $\hat{H}^0(G, \mathbb{Z}) \simeq \mathbb{Z}/\#(G)\mathbb{Z}$.