$(Q; f)$ is called a *group isotope*, if there is a group $(G; \cdot)$ and bijections α, β, γ, δ such that

$$f(x, y, z) = \delta^{-1}(\alpha x \cdot \beta y \cdot \gamma z).$$

Theorem 2. *A ternary group isotope (Q, f) belongs to $\mathfrak{P}(D_8)$ iff there exists an abelian group $(Q, +, 0)$, its involutive automorphism α and an element $a \in Q$ such that $\alpha a = -a$ and*

$$f(x_1, x_2, x_3) = \alpha x_1 + \alpha x_2 - x_3 + a.$$

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Cohomology of lattices over finite abelian groups

Andriana Plakosh

It is a joint work with Yuriy Drozd.

Let G be a finite abelian group which is a direct product of cyclic groups $C_1 \times C_2 \times \cdots \times C_s$, where $\#(C_k) = o_k$. We propose a new resolution for the trivial G-module $\mathbb{Z}$ that simplifies the calculations of cohomologies. We also study the cohomology of *G-lattices*, i.e. G-module M such that the abelian group of M is free of finite rank. We set $M^* = \mathrm{Hom}_{\mathbb{Z}}$ and $DM = \mathrm{Hom}_G(M, \mathbb{T})$, where $\mathbb{T} = \mathbb{Q}/\mathbb{Z}$. We prove the following results about Tate cohomologies $\hat{H}^n(G, M)$, where M is a G-lattice. We establish a duality theorem generalizing [**1**, Theorem XII.6.6].

Theorem 1.

$$\hat{H}^{n-1}(G, DM) \simeq D\hat{H}^{-n}(G, M),$$
$$\hat{H}^{n}(G, DM) \simeq \hat{H}^{n+1}(G, M^*),$$
$$\hat{H}^{n}(G, M^*) \simeq D\hat{H}^{-n}(G, M).$$

As $\hat{H}^n(G, M)_p \simeq \hat{H}^n(G_p, M)$, where G_p is the p-part of G, we suppose further that G is a p-group and $o_k = p^{m_k}$, where $m_1 \geq m_2 \geq \cdots \geq m_s$. Recall that a G-lattice M is called *irreducible* if so is the $\mathbb{Q}G$-module $\mathbb{Q} \otimes_{\mathbb{Z}} M$.

Set $\nu(n, s) = (-1)^n \sum_{i=0}^{n} \binom{-s}{i}$.

Theorem 2. *Let M be an irreducible G-lattice*

(1) If $M \not\simeq \mathbb{Z}$, then $\hat{H}^n(G, M) \simeq (\mathbb{Z}/p\mathbb{Z})^{\nu(|n|-1, s)}$.

(2) If $n \neq 0$, then $\hat{H}^n(G, \mathbb{Z}) \simeq \bigoplus_{k=1}^{s} (\mathbb{Z}/p^{m_k}\mathbb{Z})^{\nu(|n|-1, k) + (-1)^n}$.

Recall that $\hat{H}^0(G, \mathbb{Z}) \simeq \mathbb{Z}/\#(G)\mathbb{Z}$.

The second claim has been known [**3, 4**] but we give a new and simpler proof.

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Metric dimension of metric transform and wreath product

Bogdan Ponomarchuk

Let (X, d) be metric space. It is said that a set A is *resolving* set of the metric space (X, d) if A is non-empty subset of X and for any arbitrary different u, v from X there exists element a from A, such that distances $d(u, a)$ and $d(v, a)$ are not equal. The smallest of cardinalities of a resolving subsets of the set X is called *metric dimension* $md(X)$ of the metric space (X, d).

Definition of the metric dimension for metric spaces was firstly introduced by Blumenthal in 1953 [**1**]. 20 years later Harari and Melter [**2**] applied it to the graphs. After that the metric dimension concept found range of applications, like in combinatorial analysis, robotics, for finding its location, biology, chemistry [**3**], [**4**], [**5**].

In 2013 S. Bau and F. Beardon [**6**] got the Blumenthal's ideas and proceeded research of the metric spaces metric dimension. They has managed to calculate the metric dimension of the sphere in k-dimensional Euclidean space. Later, M. Heydarpour and S. Maghsoudi [**7**] calculated the metric dimension of geometric spaces.

As well as metric dimension, Blumenthal has also described metric transforms [**8**], which was studied further by other researchers, like by Schoenberg and von Neumann in scope of Euclidian space metric transforms into Hilbert space subsets [**9**], [**10**].

In general calculation of the metric space for graphs is NP-hard problem [**11**]. Following that, metric dimension calculation for metric spaces is also NP-hard. This why there are several ways of researching metric dimension. One of those is researching metric dimension of specific graphs constructs like wreatch product, cartesian product etc [**12**].

In this paper we will define metric dimension for the wreath product of metric spaces which was introduced by Oliynyk [**13**]. This construct of metric spaces was called wreath product because isometry group of metric spaces wreath product is isomorphic to wreath product of theirs isometry groups. In particular, for this we will show that metric dimension of metric transform of an arbitrary metric space is equal to metric dimension of this space.

Now we recall a definition of the metric dimension of metric spaces [**7**].

Let (X, d) be a metric space. It is said that a non-empty subset A of the set X resolve (X, d) provided that some element a from the subset A with $d(x, a) = d(y, a)$ it follows that $x = y$. *Metric dimension* $md(X)$ of the metric space (X, d) is called the least of cardinalities of the k resolving subsets of the set X.

Let function s, denote by $\mathbb{R}^+$ the set of all non-negative real numbers, be monotone increasing, continuous and $s(0) = 0$, such function is called *scale*. *Transformation* of metric space