The second claim has been known [**3, 4**] but we give a new and simpler proof.

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Metric dimension of metric transform and wreath product

Bogdan Ponomarchuk

Let (X, d) be metric space. It is said that a set A is *resolving* set of the metric space (X, d) if A is non-empty subset of X and for any arbitrary different u, v from X there exists element a from A, such that distances $d(u, a)$ and $d(v, a)$ are not equal. The smallest of cardinalities of a resolving subsets of the set X is called *metric dimension* $md(X)$ of the metric space (X, d).

Definition of the metric dimension for metric spaces was firstly introduced by Blumenthal in 1953 [**1**]. 20 years later Harari and Melter [**2**] applied it to the graphs. After that the metric dimension concept found range of applications, like in combinatorial analysis, robotics, for finding its location, biology, chemistry [**3**], [**4**], [**5**].

In 2013 S. Bau and F. Beardon [**6**] got the Blumenthal's ideas and proceeded research of the metric spaces metric dimension. They has managed to calculate the metric dimension of the sphere in k-dimensional Euclidean space. Later, M. Heydarpour and S. Maghsoudi [**7**] calculated the metric dimension of geometric spaces.

As well as metric dimension, Blumenthal has also described metric transforms [**8**], which was studied further by other researchers, like by Schoenberg and von Neumann in scope of Euclidian space metric transforms into Hilbert space subsets [**9**], [**10**].

In general calculation of the metric space for graphs is NP-hard problem [**11**]. Following that, metric dimension calculation for metric spaces is also NP-hard. This why there are several ways of researching metric dimension. One of those is researching metric dimension of specific graphs constructs like wreatch product, cartesian product etc [**12**].

In this paper we will define metric dimension for the wreath product of metric spaces which was introduced by Oliynyk [**13**]. This construct of metric spaces was called wreath product because isometry group of metric spaces wreath product is isomorphic to wreath product of theirs isometry groups. In particular, for this we will show that metric dimension of metric transform of an arbitrary metric space is equal to metric dimension of this space.

Now we recall a definition of the metric dimension of metric spaces [**7**].

Let (X, d) be a metric space. It is said that a non-empty subset A of the set X resolve (X, d) provided that some element a from the subset A with $d(x, a) = d(y, a)$ it follows that $x = y$. *Metric dimension* $md(X)$ of the metric space (X, d) is called the least of cardinalities of the k resolving subsets of the set X.

Let function s, denote by $\mathbb{R}^+$ the set of all non-negative real numbers, be monotone increasing, continuous and $s(0) = 0$, such function is called *scale*. *Transformation* of metric space

(X, d_X) is called space $(X, s(d_X))$, where function $s(d_X)$ might not follow triangle inequality [**8**]. Transformation is called *metric*, if $s(d_X)$ is metric.

Definition 1 ([**8**]). If for metric spaces (X, d_X) and (Y, d_Y) there is a bijection $g : X \to Y$, and scale s that for arbitrary $u, v \in X$ holds:

$$d_X(u, v) = s(d_Y(g(u), g(v))),$$

then those metric spaces are called *isomorphic.*

Proposition 1. *Let (X, d) be a metric space and let $s : \mathbb{R}^+ \to \mathbb{R}^+$ be a metric transform. Then metric basis of X is also the metric basis of the metric transform $(X, s(d))$.*

Corollary 1. *Metric dimension of X and its metric transform are equal.*

Wreath product of two simple graphs G_1 and G_2 is called a graph defined on the ordered set of pars (v, w), where v is vertex from graph G_1 and w is vertex of graph G_2. Two pairs (v, w) and (v_1, w_1) are connected by an edge if one of following conditions holds [**13**]:

(1) $v = v_1$ and there exists an edge between w and w_1 in graph G_2.
(2) there exists an edge between v and v_1 in graph G_1.

Definition 2. A metric space (X, d) is called uniformly discrete if for arbitrary $u, v \in X$ either $u = v$ or there exists radius $r > 0$ that $d(u, v) > r$.

First, we recall construction of a wreath product of metric spaces. Let (X, d_X) be a uniformly discrete metric space, and (Y, d_Y) be a bounded metric space. Since space (X, d_X) is uniformly discrete, then there exists r such that for two different arbitrary points x_1, x_2 from set X inequality $d_X(x_1, x_2) \geq r$ holds. Let $s(x)$ be the scale such that:

$$diam(s(Y)) < r \tag{1}$$

Define function ρ_s on the cartesian product $X \times Y$ as:

$$\rho_s((x_1, y_1), (x_2, y_2)) = \begin{cases} d_X(x_1, x_2), & \text{if } x_1 \neq x_2 \\ s(d_Y(y_1, y_2)), & \text{if } x_1 = x_2 \end{cases}$$

Such metric space is called *wreath product* of metric spaces (X, d_X) and (Y, d_Y) and denoted as $XwrY$ [**14**]

Theorem 1. *Let X be a finite metric space and Y be a bounded metric space. If $dimY < \infty$ then dimension of wreath product of metric spaces (X, d_X) and (Y, d_Y) is equal to*

$$dim(Xwr_sY) = ||X|| * dim(Y)$$

If $dimY = \infty$, then $dim(Xwr_sY) = \infty$.

Proposition 2. *If space X is infinite, then $dimXwr_sY = \infty$.*

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On some LCA groups with commutative π-regular ring of continuous endomorphisms

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Let $\mathcal{L}$ be the class of locally compact abelian groups. For $X \in \mathcal{L}$, let $E(X)$ denote the ring of all continuous endomorphisms of X. If $X \in \mathcal{L}$ is topologically torsion, let $S(X) = \{p \mid p \text{ is a prime and } X_p \neq \{0\}\}$, where $X_p = \{x \in X \mid \lim_{n\to\infty} p^n x = 0\}$. We denote by $\mathbb{Q}$ the group of rationals with the discrete topology, by $\mathbb{Q}^*$ the caracter group of $\mathbb{Q}$, and by $\mathbb{R}$ the group of reals, both taken with their usual topologies. Given a prime p and a positive integer n, we denote by $\mathbb{Z}(p^n)$ the discrete group of integers modulo p^n, by $\mathbb{Z}_p$ the group of p-adic integers, and by $\mathbb{Q}_p$ the group of p-adic numbers, both endowed with their usual topologies. Also, if $(X_i)_{i\in I}$ is a family of groups in $\mathcal{L}$ and, for each $i \in I$, U_i is a compact open subgroup of X_i, then $\prod_{i\in I}(X_i; U_i)$ denotes the local direct product of the groups X_i with respect to the subgroups U_i.

Theorem 1. *Let X be a non-residual group in $\mathcal{L}$. The ring $E(X)$ is commutative and π-regular if and only if X is topologically isomorphic with one of the groups:*

$$\mathbb{R} \times \prod_{p\in S(_1)} \big(\mathbb{Z}(p^{n_p}); k_p\mathbb{Z}(p^{n_p})\big) \times \prod_{p\in S_2} \big(\mathbb{Q}_p; \mathbb{Z}_p\big) \times \prod_{p\in S_3} \big(\mathbb{Q}_p \times \mathbb{Z}(p^{n_p}); \mathbb{Z}_p \times k_p\mathbb{Z}(p^{n_p}),\big)$$

$$\mathbb{Q} \times \prod_{p\in S(X)} \big(\mathbb{Z}(p^{n_p}); k_p\mathbb{Z}(p^{n_p})\big), \quad \text{and} \quad \mathbb{Q}^* \times \prod_{p\in S(X)} \big(\mathbb{Z}(p^{n_p}); k_p\mathbb{Z}(p^{n_p})\big),$$

where $S_1 \cup S_2 \cup S_3 = S(X)$, $S_i \cap S_j = \varnothing$ for all $i \neq j$, and the n_p's and k_p's are non-negative integers.

Theorem 2. *Let $X \in \mathcal{L}$ be torsion-free and topologically completely decomposable. The ring $E(X)$ is commutative and π-regular if and only if either X is discrete and the rank one components of its decomposition have pairwise incomparable types, or X is topologically isomorphic with either $\mathbb{Q}^*$ or $\prod_{p\in S(X)}\big(\mathbb{Q}_p; \mathbb{Z}_p,\big)$.*

Theorem 3. *Let $X \in \mathcal{L}$ be densely divisible and topologically completely decomposable. The ring $E(X)$ is commutative and π-regular if and only if either X is compact and the rank one components of its decomposition have pairwise incomparable patterns, or X is topologically isomorphic with either $\mathbb{Q}$ or $\prod_{p\in S(X)}\big(\mathbb{Q}_p; \mathbb{Z}_p,\big)$.*