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Key words and phrases. Metric space, metric dimension, metric transforms, wreath product

On some LCA groups with commutative π-regular ring of continuous endomorphisms

Valeriu Popa

Let $\mathcal{L}$ be the class of locally compact abelian groups. For $X \in \mathcal{L}$, let $E(X)$ denote the ring of all continuous endomorphisms of X. If $X \in \mathcal{L}$ is topologically torsion, let $S(X) = \{p \mid p$ is a prime and $X_p \neq \{0\}\}$, where $X_p = \{x \in X \mid \lim_{n\to\infty} p^n x = 0\}$. We denote by $\mathbb{Q}$ the group of rationals with the discrete topology, by $\mathbb{Q}^*$ the caracter group of $\mathbb{Q}$, and by $\mathbb{R}$ the group of reals, both taken with their usual topologies. Given a prime p and a positive integer n, we denote by $\mathbb{Z}(p^n)$ the discrete group of integers modulo p^n, by $\mathbb{Z}_p$ the group of p-adic integers, and by $\mathbb{Q}_p$ the group of p-adic numbers, both endowed with their usual topologies. Also, if $(X_i)_{i\in I}$ is a family of groups in $\mathcal{L}$ and, for each $i \in I$, U_i is a compact open subgroup of X_i, then $\prod_{i\in I}(X_i; U_i)$ denotes the local direct product of the groups X_i with respect to the subgroups U_i.

Theorem 1. *Let X be a non-residual group in $\mathcal{L}$. The ring $E(X)$ is commutative and π-regular if and only if X is topologically isomorphic with one of the groups:*

$$\mathbb{R} \times \prod_{p\in S(_1)} \big(\mathbb{Z}(p^{n_p}); k_p\mathbb{Z}(p^{n_p})\big) \times \prod_{p\in S_2} \big(\mathbb{Q}_p; \mathbb{Z}_p\big) \times \prod_{p\in S_3} \big(\mathbb{Q}_p \times \mathbb{Z}(p^{n_p}); \mathbb{Z}_p \times k_p\mathbb{Z}(p^{n_p}),\big)$$

$$\mathbb{Q} \times \prod_{p\in S(X)} \big(\mathbb{Z}(p^{n_p}); k_p\mathbb{Z}(p^{n_p})\big), \quad \text{and} \quad \mathbb{Q}^* \times \prod_{p\in S(X)} \big(\mathbb{Z}(p^{n_p}); k_p\mathbb{Z}(p^{n_p})\big),$$

where $S_1 \cup S_2 \cup S_3 = S(X)$, $S_i \cap S_j = \varnothing$ for all $i \neq j$, and the n_p's and k_p's are non-negative integers.

Theorem 2. *Let $X \in \mathcal{L}$ be torsion-free and topologically completely decomposable. The ring $E(X)$ is commutative and π-regular if and only if either X is discrete and the rank one components of its decomposition have pairwise incomparable types, or X is topologically isomorphic with either $\mathbb{Q}^*$ or $\prod_{p\in S(X)}\big(\mathbb{Q}_p; \mathbb{Z}_p,\big)$.*

Theorem 3. *Let $X \in \mathcal{L}$ be densely divisible and topologically completely decomposable. The ring $E(X)$ is commutative and π-regular if and only if either X is compact and the rank one components of its decomposition have pairwise incomparable patterns, or X is topologically isomorphic with either $\mathbb{Q}$ or $\prod_{p\in S(X)}\big(\mathbb{Q}_p; \mathbb{Z}_p,\big)$.*

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Key words and phrases. LCA groups, rings of continuous endomorphisms

Finite field elements of high order on a base of modified Gao approach

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Elements of high multiplicative order in finite fields are of great interest in several applications (cryptography, error correcting codes) that use finite fields. Obviously, the best possible are primitive elements, but there is no any algorithm to find them. Therefore, they consider a less ambitious question: to find an element with provable high order [**3, 4**].

F_{q^n} is a field with q^n elements, where q is a power of a prime number p and n is an integer. u is the nearest larger integer to $\log_q n$.

Gao [**3**] described construction of high order elements for general extensions F_{q^n} of finite field F_q . For this goal, he searched for a polynomial $g(x) \in F_q[x]$ of small degree such that $x^{q^u} - g(x)$ has irreducible factor $f(x)$ of degree n. The method was improved in [**1, 2, 5**].

The modification is as follows [**1, 6**]: to search for polynomials $g(x), h(x) \in F_q[x]$ of small degrees such that $h(x)x^{q^u} - g(x)$ has an irreducible divisor $f(x)$ of degree n. However, the bound on the order was not improved.

We have performed calculations in Maple and obtained examples, which show that the modified Gao approach can give better lower bound on the order.

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Key words and phrases. Finite field, multiplicative order, lower bound