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Algebraic structures in the theory of fractals (fractal geometry and fractal analysis)

Mykola Pratsiovytyi

The fractal is a set of complete metric space that have the property of self-similarity in some sense (classic self-similarity, self-affinity or "structural similarity") or have fractional metric dimension (like a Hausdorff–Besicovitch or Minkowski dimension, entropic or box-counting dimension) as well as such that its metric and topological dimension are unequal.

The publication of Felix Hausdorff's paper "Dimension und äußeres Maß", Math. Ann. **79** (1919), 157–179, should be considered as the birth of the theory of fractals. Hausdorff introduced the definition of fractal set and initiated the study of such sets using the so-called Hausdorff measures constructed by the C. Carathéodory principle. Before this, there was some interest to various sets (figures), which are called the simplest (self-similar) fractals today. Cantor set, Sierpiński carpets and Koch snowflake are among them. Thus, 100th anniversary of the theory of fractals is celebrated this year. The results of A. S. Besicovitch, P. Billingsley, H. G. Eggleston, John E. Hutchinson and other authors were stages in the history of development of this theory. The interest to the theory of fractals has increased after publication of Benoit B. Mandelbrot's book as well as various monographs and handbooks devoted to fractals and their applications.

Today fractals appear in different fields of mathematics. Many objects of continuous mathematics have fractal properties, namely, objects such that their essential sets have fractal structure. Functions, measures, transformations of space, dynamical systems etc. are among them. Some of such objects are directly related or induced by algebraic structures in the phase space. Some representatives of groups of similarity transformations, groups of affine transformations, and linear spaces can be used for construction of systems of encoding of real numbers and for development of the corresponding analytic theory for definition and studying of fractal objects. We understand the fractality of these objects in different ways.

The talk is devoted to functions, probability measures and transformations of Euclidean space. They have sets of various peculiarities with fractal structure, supports of distribution and attractors of dynamical systems as well as transformations defined by invariants related to fractal characteristics, dimension, self-similarity, etc. Systems of self-similar representations, functions preserving frequency of digits of representation, their mean values, tails of representation etc. as well as measures with self-similar essential supports of distribution are studied in detail.

We focus on the normal properties of numbers (on the base of Lebesgue measure). For systems of representation with constant and variable alphabet, problems of probabilistic theory of numbers defined by the system of invariants of representation are also discussed.

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Mathematical structures with fractal properties in the space of sequences of zeros and ones

MYKOLA PRATSIOVYTYI, SOFIIA RATUSHNIAK

Let $A = \{0, 1\}$ be an alphabet and let $L \equiv A \times A \times A \times ... \times A \times ...$ be a space of sequences of zeros and ones. An element of this space is denoted by a (a_n) or $\overline{a}$.

A mapping f of space L onto a set $[0, 1]$ given by a formula

$$L \ni (\alpha_n) \xrightarrow{f} x = \frac{\alpha_1}{2} + \frac{\alpha_2}{2^2} + ... + \frac{\alpha_n}{2^n} + ... \equiv \Delta^2_{\alpha_2\alpha_2...\alpha_n...}$$

is called a classical binary representation.

The binary representation of numbers is a useful tool to develop binary analysis, metric and probabilistic theory of numbers. The simplicity of geometry of this representation generates various applications in the theory of fractals. It is instrument for studying functions with various fractal properties and complicated local structure (behaviour).

The metrization of space L by means of binary representation and the function

$$f_1(\overline{x}, \overline{y}) = \sum_{k=1}^{\infty} \frac{|x_k - y_k|}{2^k}$$

allows us to develop a theory of fractals in the space L, which has own features and differences as compared with the Euclidean metric in $[0, 1]$.

Let $E \subset L$ be a set of all sequences such that frequencies of digits 0 and 1 are equal to $p_0 \neq 0$ and $p_1 = 1 - p_0 \neq 0$ respectively. In this set, we consider the metric

$$f_2(\overline{p}, \overline{q}) = |\ln \frac{p(1-q)}{q(1-p)}|,$$

where $\overline{p} \equiv (p, 1-p)$, $\overline{q} \equiv (q, 1-q)$, and fractal subsets defined by different conditions.

In the metric space (L, f_2) we develop theory of fractal sets and others objects with fractal properties.

In our talk we develop fractal analysis in the space L by using different two-symbol representations of real numbers, which are generalizations of classical binary representation (Q_2^*-representation) or its analogues (A_2-continued fraction representation, etc.). Self-similar and non-self-similar representations with zero and non-zero redundancy are among them.

Functions with fractal properties and dynamical systems with locally complicated mappings are studied in detail. In particular, we discuss properties of distribution of values of function $f_\varphi(\Delta^{Q_2^*}_{\alpha_1\alpha_2...\alpha_n...}) = \Delta^{Q_2^*}_{\varphi_1(\alpha_1,\alpha_2)\varphi_2(\alpha_2,\alpha_3)\varphi_n(\alpha_n,\alpha_{n+1})...}$, where (φ_n) is a sequence of finite functions that are defined on a four-element set $A^2 \equiv A \times A$ and take values from the set A; and $\Delta^{Q_2^*}_{\alpha_1\alpha_2...\alpha_n...}$ is the Q_2^*-representation of number $x \in [0, 1]$.