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On similarity of tuples of matrices over a field

Volodymyr Prokip

Let $\mathbb{F}$ be a field. Denote by $\mathbb{F}_{m\times n}$ the set of $m \times n$ matrices over $\mathbb{F}$ and by $\mathbb{F}_{m\times n}[x_1, x_2, , x_n]$ the set of $m \times n$ matrices over the polynomial ring $\mathbb{F}[x_1, x_2, , x_n]$. In what follows, we denote by I_n the $n \times n$ identity matrix and by $0_{n,k}$ the zero $m \times n$ matrix. The Kronecker product of matrices $A = \left[\ a_{ij}\ \right] \in \mathbb{F}_{m\times n}$ and B is denoted by $A \otimes B = \left[\ a_{ij}B\ \right]$.

Two tuples of $n \times n$ matrices $\mathbf{A} = \{A_1, A_2, \ldots, A_k\}$ and $\mathbf{B} = \{B_1, B_2, \ldots, B_k\}$ over a field $\mathbb{F}$ are said to be simultaneously similar if there exists a nonsingular matrix $U \in \mathbb{F}_{n\times n}$ such that $A_i = U^{-1}B_iU$ for all $i = 1, 2, \ldots, k$. The task of classifying square matrices up to similarity is one of the core and oldest problems in linear algebra (see [**1**], [**2**], [**3**] and references therein), and it is generally acknowledged that it is also one of the most hopeless problems already for $k = 2$. For given matrices $A_i, B_i \in \mathbb{F}_{n\times n}$ we define matrices

$$M_i = \left[\ A_i \otimes I_n - I_n \otimes B_i^T\ \right] \in \mathbb{F}_{n^2\times n^2},\ i = 1, 2, \ldots, k; \quad \text{and} \quad M = \begin{bmatrix} M_1 \\ M_2 \\ \vdots \\ M_k \end{bmatrix} \in \mathbb{F}_{kn^2\times n^2}.$$

Theorem 1. *If two tuples of $n\times n$ matrices $\mathbf{A} = \{A_1, A_2, \ldots, A_k\}$ and $\mathbf{B} = \{B_1, B_2, \ldots, B_k\}$ over a field $\mathbb{F}$ are simultaneously similar then* $\operatorname{rank} M < n^2$.

Let $\operatorname{rank} M = n^2 - r$, where $r \in \mathbb{N}$. For the matrix M there exists a nonsingular matrix $U \in \mathbb{F}_{n^2\times n^2}$ such that $MU = \left[\ H \quad 0_{kn^2,r}\ \right]$, where $H \in \mathbb{F}_{kn^2\times(n^2-r)}$. Put $U = \left[\ U_1 \quad U_2\ \right]$, where $U_2 \in \mathbb{F}_{n^2\times r}$. For independent variables $x_1, x_2, \ldots, x_r$ we construct the vector

$$U_2 \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_r \end{bmatrix} = \begin{bmatrix} V_1(\overline{x}) \\ V_2(\overline{x}) \\ \vdots \\ V_n(\overline{x}) \end{bmatrix}, \quad \text{where} \quad V_i(\overline{x}) = V_i(x_1, \ldots, x_r) \in \mathbb{F}_{n,1}[x_1, x_2, \ldots, x_r].$$

Theorem 2. *Two tuples of $n\times n$ matrices $\mathbf{A} = \{A_1, A_2, \ldots, A_k\}$ and $\mathbf{B} = \{B_1, B_2, \ldots, B_k\}$ over a field $\mathbb{F}$ of characteristic 0 are simultaneously similar if and only if the matrix* $\begin{bmatrix} V_1^T(\overline{x}) \\ V_2^T(\overline{x}) \\ \cdots \\ V_n^T(\overline{x}) \end{bmatrix}$ *is nonsingular.*

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Extensions of finite fields and some class of special p-groups

Olga Pyliavska

A finite p-group G is called special if the center $Z(G)$, the commutator subgroup G' and the Frattini subgroup $\Phi(G)$ coincide ([**4**]).

Special p-groups have nilpotency class 2. For these groups $Z(G)$ and G/G' are elementary abelian and exponent of G is p or p^2.

The special p-groups of exponent p admit some matrix presentation over the field $F_p = \mathbb{Z}/p\mathbb{Z}$ (see [**1**], [**5**], [**6**]), which gives possibility for their classification.

We define some class of special p-groups of exponent $\leq p^2$ which admit the calculation in the extension of F_{p^n} of finite field F_p. The groups of investigation has order p^{3n}, where $n = gcd(n, p-1)$ and $|G'| = p^n$.

For small n and arbitrary prime p are obtained

- full classification of these groups up to isomorphism and their enumeration;
- the structure of maximal abelian normal subgroups and corresponding factor-groups;
- automorphism groups.

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