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Some special p-groups and nearrings with identity

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Clearly every associative ring is a nearring and each group is the additive group of a nearring, but not necessarily of a nearring with identity. The question what group can be the additive group of a nearring with identity is far from solution.

We investigate p-groups with cyclic subgroup of index p as the additive groups of nearrings with identity.

In [**1**, Theorem 12.5.1] it was proved that there exist seven types of p-groups with cyclic subgroup of index p.

Theorem 1. *Let G be a group from* [**1**, Theorem 12.5.1]. *G is the additive group of a nearring with identity iff one of the following statement holds:*

(1) $G = \langle a |\ a^{p^n} = 1\rangle$, $n \geq 1$.
(2) $G = \langle a,\ b |\ a^{p^{n-1}} = 1,\ b^p = 1,\ ba = ab\rangle$, $n \geq 2$.
(3) $G = \langle a,\ b |\ a^{p^{n-1}} = 1,\ b^p = 1,\ ba = a^{1+p^{n-2}}b\rangle$, *$p$ is odd*, $n \geq 3$.
(4) G is a dihedral group of order 8.
(5) $G = \langle a,\ b |\ a^{2^{n-1}} = 1,\ b^2 = 1,\ ba = a^{1+2^{n-2}}b\rangle$, $n \geq 4$.

Denote by $n(G)$ the number of all non-isomorphic zero-symmetric nearrings with identity whose additive group R^+ is isomorphic to the group G.

So using [**3**, Theorem 7.1] and [**2**] we can easily conclude the following result:

Proposition 1. *Let G be a non-abelian group from Theorem 1. Then the following statements hold:*

(1) If $p = 2$ *and* $n = 3$, *then* $n(G) = 7$.
(2) If $p = 2$ *and* $n = 4$, *then* $n(G) = 32$.
(3) If $p = 2$ *and* $n > 4$, *then* $n(G) = 2^{n+2}$.
(4) If $p = 3$, *then* $n(G) = 3^{n-2}$.
(5) If $p > 3$, *then* $n(G) = p^{n-3}$.

References

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Reduction of nonsingular matrices over rings of almost stable range 1

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All rings consider of will be a commutative with nonzero units. Recall that a ring R is a Bezout ring if it every finitely generated ideal is principal. A ring R is called an elementary divisor ring if for any $n \times m$ matrix A over R there exist invertible matrices $P \in GL_n(R)$ and $Q \in GL_m(R)$ such that $PAQ = D$ is a diagonal matrix. $D = (d_{ii})$ and $d_{i+1,i+1}R \subseteq d_{ii}R$ [**1**].

We denote by GE_n the subgroup of $GL_n(R)$ generated by the elementary matrices.

A ring R is called a ring of stable range 1 if for any elements $a, b \in R$ the equality $aR+bR = R$ implies that there is some $x \in R$ such that $(a + bx)R = R$.

Definition 1. An element $a \neq 0$ of a commutative ring R is called an element almost stable range 1 if the stable range of a factor-ring R/aR is equal to 1. If all nonzero elements of a ring R are elements of almost stable range 1 then we say that R is a ring of almost stable range 1.

Theorem 1. *Let R be commutative Bezout domain of almost stable range 1, then for any nonsingular matrix of size n we can find such unimodular matrices $P \in GE_n(R)$ and $Q \in GL_n(R)$, that*

$$PAQ = \begin{pmatrix} \varepsilon_1 & 0 & \dots & 0 \\ 0 & \varepsilon_2 & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \dots & \varepsilon_n \end{pmatrix},$$

where ε_i is divisor ε_{i+1}, $1 \leq i \leq n-1$.

References

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