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Key words and phrases. p-Group, nearring with identity

Reduction of nonsingular matrices over rings of almost stable range 1

Andriy Romaniv

All rings consider of will be a commutative with nonzero units. Recall that a ring R is a Bezout ring if it every finitely generated ideal is principal. A ring R is called an elementary divisor ring if for any $n \times m$ matrix A over R there exist invertible matrices $P \in GL_n(R)$ and $Q \in GL_m(R)$ such that $PAQ = D$ is a diagonal matrix. $D = (d_{ii})$ and $d_{i+1,i+1}R \subseteq d_{ii}R$ [**1**].

We denote by GE_n the subgroup of $GL_n(R)$ generated by the elementary matrices.

A ring R is called a ring of stable range 1 if for any elements $a, b \in R$ the equality $aR+bR = R$ implies that there is some $x \in R$ such that $(a + bx)R = R$.

Definition 1. An element $a \neq 0$ of a commutative ring R is called an element almost stable range 1 if the stable range of a factor-ring R/aR is equal to 1. If all nonzero elements of a ring R are elements of almost stable range 1 then we say that R is a ring of almost stable range 1.

Theorem 1. *Let R be commutative Bezout domain of almost stable range 1, then for any nonsingular matrix of size n we can find such unimodular matrices $P \in GE_n(R)$ and $Q \in GL_n(R)$, that*

$$PAQ = \begin{pmatrix} \varepsilon_1 & 0 & \dots & 0 \\ 0 & \varepsilon_2 & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \dots & \varepsilon_n \end{pmatrix},$$

where ε_i is divisor ε_{i+1}, $1 \leq i \leq n-1$.

References

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Key words and phrases. Reduction of nonsingular matrices, rings almost stable range 1

On the conjugate sets of IP-quasigroups

Tatiana Rotari

A quasigroup (Q, A) is called quasigroup with the property of invertibility *(an IP-quasigroup)* if there exist two mappings I_l an I_r on the set Q into Q that $A(I_l x,\, A(x,\, y)) = y$ and $A(A(y,\, x),\, I_r x) = y$ for any $x, y \in Q$ [1]. The mappings I_l and I_r are permutations and $I_l^2 = I_r^2 = \varepsilon$.

It is known that the system Σ of six (not necessarily distinct) conjugates (or parastrophes): A, ${}^{r}A$, ${}^{l}A$, ${}^{rl}A$,${}^{lr}A$,${}^{s}A$, where ${}^{r}A(x,y) = z \Leftrightarrow A(x,z) = y$, ${}^{l}A(x,y) = z \Leftrightarrow A(z,y) = x$, ${}^{s}A(x,y) = A(y,x)$ (${}^{rl}A = {}^{r}({}^{l}A)$) corresponds to a quasigroup (Q, A).

It is known [**2**] that the number of distinct conjugates in Σ can be 1, 2, 3 or 6.

Using suitable Belousov's designation of conjugates of a quasigroup (Q, A) from [**1**] we have the following system Σ of conjugates:

$$\Sigma = \left\{A,\ {}^{r}A,\ {}^{l}A,\ {}^{lr}A,\ {}^{rl}A,\ {}^{s}A\right\},$$

where ${}^{\iota}A = A$, ${}^{r}A = A^{-1}$, ${}^{l}A = {}^{-1}A$, ${}^{lr}A = {}^{-1}(A^{-1})$, ${}^{rl}A = ({}^{-1}A)^{-1}$,${}^{s}A = A^{*}$.

Note that

$$\left({}^{-1}(A^{-1})\right)^{-1} = {}^{rlr}A = {}^{-1}\left(({}^{-1}A)^{-1}\right) = {}^{lrl}A = {}^{s}A$$

and ${}^{rr}A = {}^{ll}A = A$, ${}^{\sigma\tau}A = {}^{\sigma}({}^{\tau}A)$.

The conjugates og IP-quasigroup have the following form [1, 4]:

$${}^{l}A(x,\, y) = A(x,\, I_r y),\ {}^{r}A(x,\, y) = A(I_l x,\, y),\ {}^{lr}A(x,\, y) = I_l A(x,\, I_l y),$$

$${}^{rl}A(x,\, y) = I_r A(I_l x,\, y),\ {}^{s}A(x,\, y) = I_l A(I_r x,\, I_r y).$$

The following Theorem 1 of [**3, 4**] describes all possible conjugate sets for quasigroups and points out the only possible variants of equality of conjugates:

Theorem 1. *The following conjugate sets of a quasigroups $(Q,\, A)$ are only possible:* $\overline{\Sigma}_1(A) = \{A\}$, $\overline{\Sigma}_2 = \{A,\, {}^{s}A\} = \{A = {}^{lr}A = {}^{rl}A,\, {}^{l}A = {}^{r}A = {}^{s}A\}$, $\overline{\Sigma}_6 = \{A,\, {}^{l}A,\, {}^{r}A,\, {}^{lr}A,\, {}^{rl}A,\, {}^{s}A\}$, $\overline{\Sigma}_3 = \{A,\, {}^{lr}A,\, {}^{rl}A\}$ *and three cases are only possible:* $\overline{\Sigma}_3^1 = \{A = {}^{r}A,\, {}^{l}A = {}^{lr}A,\, {}^{rl}A = {}^{s}A\}$; $\overline{\Sigma}_3^2 = \{A = {}^{l}A,\, {}^{r}A = {}^{rl}A,\, {}^{lr} = {}^{s}A\}$; $\overline{\Sigma}_3^3 = \{A = {}^{s}A,\, {}^{r}A = {}^{lr}A,\, {}^{l}A = {}^{rl}A\}$.

We study the conjugate sets on the distict conjugates of IP-quasigroups and IP-loops.

Theorem 2. *Let a quasigroup $(Q,\, A)$ be an IP-quasigroup. Then*
$\Sigma(A) = \overline{\Sigma}_1(A)$ *if and only if* $I_r = I_l = I = \varepsilon$;
$\Sigma(A) = \overline{\Sigma}_2(A)$ *if and only if* $I_l = I_r = I \neq \varepsilon$, $A(x,\, y) \neq A(y,\, x)$ *and* $IA(x,\, y) = A(y,\, x)$;
$\Sigma(A) = \overline{\Sigma}_3^1(A)$ *if and only if* $I_l = \varepsilon \neq I_r$;
$\Sigma(A) = \overline{\Sigma}_3^2(A)$ *if and only if* $I_r = \varepsilon \neq I_l$;
$\Sigma(A) = \overline{\Sigma}_3^3(A)$ *if and only if* $I_l = I_r = I \neq \varepsilon$ *and* $A(x,\, y) = A(y,\, x)$;
$\Sigma(A) = \overline{\Sigma}_6(A)$ *if and only if* $I_l = I_r = I \neq \varepsilon$, $A(x,\, y) \neq A(y,\, x)$ *and* $IA(x,\, y) \neq A(y,\, x)$.;