A special case of IP-loops is a Moufang loop defined by the identity

$$A(x,\ A(y,\ A(x,\ z))) = A(A(A(x,\ y), x), z).$$

From the Theoreme the following corollary easy follow.

Corollary 1. *Let (Q, A) be an IP-loop (a Moufang loop), then*
$\Sigma(A) = \overline{\Sigma}_1(A)$ *if and only if* $I = \varepsilon$;
$\Sigma(A) = \overline{\Sigma}_3^3(A)$ *if and only if* $(Q,\ A)$ *is commutative and* $I \neq \varepsilon$;
$\Sigma(A) = \overline{\Sigma}_6(A)$ *if and only if* $(Q,\ A)$ *is noncommutative.*

Note that the case $\Sigma(A) = \overline{\Sigma}_2(A)$ ($\overline{\Sigma}_3^1(A)$ or $\overline{\Sigma}_3^2(A)$) for any IP-loops is impossible.

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On sublattices of the lattice of multiply saturated formations of finite groups

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All groups considered are finite. We use terminology and notations from [**1**]–[**3**].

Let σ be some partition of the set of all primes $\mathbb{P}$, that is, $\sigma = \{\sigma_i | i \in I\}$, where $\mathbb{P} = \bigcup_{i\in I} \sigma_i$ and $\sigma_i \cap \sigma_j = \emptyset$ for all $i \neq j$; $\Pi \subseteq \sigma$ and $\Pi' = \sigma \setminus \Pi$. If n is an integer, the symbol $\sigma(n)$ denotes the set $\{\sigma_i | \sigma_i \cap \pi(n) \neq \emptyset\}$, if G is a finite group, then $\sigma(G) = \sigma(|G|)$, and if $\mathfrak{F}$ is a class of groups, then $\sigma(\mathfrak{F}) = \bigcup_{G\in\mathfrak{F}} \sigma(G)$.

A function f of the form $f : \sigma \to \{\text{formations of groups}\}$ is called *a formation σ-function.* For any formation σ-function f the symbol $LF_\sigma(f)$ denotes the class

$$LF_\sigma(f) = (G \mid G = 1 \text{ or } \ G \neq 1 \text{ and } \ G/O_{\sigma_i',\sigma_i}(G) \in f(\sigma_i) \text{ for all } \sigma_i \in \sigma(G)).$$

If for some formation σ-function f we have $\mathfrak{F} = LF_\sigma(f)$, then we say, that the class $\mathfrak{F}$ is *σ-local* and f is a *σ-local definition of* $\mathfrak{F}$.

We suppose that every formation of groups is *0-multiply σ-local*; for $n \geq 1$, we say that the formation $\mathfrak{F}$ is *n-multiply σ-local* provided either $\mathfrak{F} = (1)$ is the formation of all identity groups or $\mathfrak{F} = LF_\sigma(f)$, where $f(\sigma_i)$ is $(n-1)$-multiply σ-local for all $\sigma_i \in \sigma(\mathfrak{F})$. The formation $\mathfrak{F}$ is said to be *totally σ-local* provided $\mathfrak{F}$ it is n-multiply σ-local for all $n \in \mathbb{N}$.

In the classical case, when $\sigma = \sigma^1 = \{\{2\}, \{3\}, \dots\}$, a formation σ-function, a σ-local formation and an n-multiply σ-local formation are, respectively, a formation function, a local formation (a saturated formation), and an n-multiply local formation (an n-multiply saturated formation) in the usual sense [**4**]–[**6**].

As shown in [**3**] the set $\mathcal{S}_n^\sigma$ of all n-multiply σ-local formations forms a complete algebraic modullar lattice.

Theorem 1. *The lattice $\mathcal{S}_n^\sigma$ of all n-multiply σ-local formations is a complete sublattice of the lattice of all n-multiply saturated formations.*

In the case when $n = 1$, we get from Theorem 1 the following resalt.

Corollary 1. *The lattice $\mathcal{S}^\sigma$ of all σ-local formations is a complete sublattice of the lattice of all saturated formations.*

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Elementary reduction of idempotent matrices over semiabelian rings

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A ring R is a associative ring with nonzero identity. An elementary $n \times n$ matrix with entries from R is a square $n \times n$ matrix of one of the types below:
1) diagonal matrix with invertible diagonal entries;
2) identity matrix with one additional non diagonal nonzero entry;
3) permutation matrix, i.e. result of switching some columns or rows in the identity matrix.

A ring R is called a ring with elementary reduction of matrices in case of an arbitrary matrix over R possesses elementary reduction, i.e.for an arbitrary matrix A over the ring R there exist such elementary matrices over R, $P_1, \ldots, P_k, Q_1, \ldots, Q_s$ of respectful size that

$$P_1 \cdots P_k \cdot A \cdot Q_1 \cdots Q_s = diag(\varepsilon_1, \ldots, \varepsilon_r, 0, \ldots, 0),$$

where $R\varepsilon_{i+1}R \subseteq R\varepsilon_i \cap \varepsilon_i R$ for any $i = 1, \ldots, r-1$.