As shown in [**3**] the set $\mathcal{S}_n^\sigma$ of all n-multiply σ-local formations forms a complete algebraic modullar lattice.

Theorem 1. *The lattice $\mathcal{S}_n^\sigma$ of all n-multiply σ-local formations is a complete sublattice of the lattice of all n-multiply saturated formations.*

In the case when $n = 1$, we get from Theorem 1 the following resalt.

Corollary 1. *The lattice $\mathcal{S}^\sigma$ of all σ-local formations is a complete sublattice of the lattice of all saturated formations.*

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Contact information

Inna N. Safonova
Department of Mathematics and Mechanics, Belarusian State University, Minsk, Belarus
Email address: safonova@bsu.by

Vasily G. Safonov
Department of Mathematics and Mechanics, Belarusian State University, Minsk, Belarus
Email address: vgsafonov@bsu.by

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Elementary reduction of idempotent matrices over semiabelian rings

Andrii Sahan

A ring R is a associative ring with nonzero identity. An elementary $n \times n$ matrix with entries from R is a square $n \times n$ matrix of one of the types below:
1) diagonal matrix with invertible diagonal entries;
2) identity matrix with one additional non diagonal nonzero entry;
3) permutation matrix, i.e. result of switching some columns or rows in the identity matrix.

A ring R is called a ring with elementary reduction of matrices in case of an arbitrary matrix over R possesses elementary reduction, i.e.for an arbitrary matrix A over the ring R there exist such elementary matrices over R, $P_1, \ldots, P_k, Q_1, \ldots, Q_s$ of respectful size that

$$P_1 \cdots P_k \cdot A \cdot Q_1 \cdots Q_s = diag(\varepsilon_1, \ldots, \varepsilon_r, 0, \ldots, 0),$$

where $R\varepsilon_{i+1}R \subseteq R\varepsilon_i \cap \varepsilon_i R$ for any $i = 1, \ldots, r-1$.

A ring R is called EID-ring in case of an indempotent matrix over R possesses elementary-idempotent reduction, i.e.for an indempotent matrix A over the ring R there exist such elementary matrices over R, $U_1, \ldots, U_l$ of respectful size that

$$U_1 \cdots U_l \cdot A \cdot (U_1 \cdots U_l)^{-1} = diag(d_1, d_2, \ldots, d_r, 0, \ldots, 0),$$

where $l, r \in \mathbb{N}$.

An idempotent e in a ring R is called right (left) semicentral if for every $x \in R$, $ex = exe$ ($xe = exe$). And the set of right (left) semicentral idempotents of R is denoted by $S_r(R)$ ($S_l(R)$). We define a ring R semiabelian if $Id(R) = S_r(R) \cup S_l(R)$.

All other necessary definitions and facts can be found in [1, 2, 3].

Theorem 1. *Let R be an semiabelian ring and A be an $n \times n$ idempotent matrix over R. If there exist elementary matrices $P_1, \ldots, P_k$ and $Q_1, \ldots, Q_s$ such that $P_1 \cdots P_k \cdot A \cdot Q_1 \cdots Q_s$ is a diagonal matrix, then there is elementary matrices $U_1, \ldots, U_l$ such that $U_1 \cdots U_l \cdot A \cdot (U_1 \cdots U_l)^{-1}$ is diagonal matrix.*

Theorem 2. *Let R be an semiabelian ring. Then a ring with elementary reduction of matrices is an EID-ring.*

Theorem 3. *The following are equivalent for a semialelian ring R:*

(a) *Each idempotent matrix over R is diagonalizable under a elementary transformation.*
(b) *Each idempotent matrix over R has a charateristic vector.*

Theorem 4. *Let R be an semiabelian ring, N be the set of nilpotents in R, and I be an ideal in R with $I \subseteq N$. Then R/I is an EID-ring, if and only if R is an EID-ring.*

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Contact information

Andrii Sahan
Department of Mechanics and Mathematics, Ivan Franko National University of Lviv, Lviv, Ukraine
Email address: andrijsagan@gmail.com

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Higher power moments of the Riesz mean error term of hybrid symmetric square L-function

Olga Savastru

Let $f(z) = \sum_{n=1}^{\infty} a_f(n) e^{2\pi i n z}$ be a holomorphic cusp form of even weight $k \geq 12$ for the full modular group $SL(2, \mathbb{Z})$, $z \in H$, $H = \{z \in \mathbb{C} | Im(z) > 0\}$ is the upper half plane. We suppose that $f(z)$ is a normalized eigenfunction for the Hecke operators $T(n)(n \geq 1)$ with $a_f(1) = 1$.