A ring R is called EID-ring in case of an indempotent matrix over R possesses elementary-idempotent reduction, i.e.for an indempotent matrix A over the ring R there exist such elementary matrices over R, $U_1, \ldots, U_l$ of respectful size that

$$U_1 \cdots U_l \cdot A \cdot (U_1 \cdots U_l)^{-1} = diag(d_1, d_2, \ldots, d_r, 0, \ldots, 0),$$

where $l, r \in \mathbb{N}$.

An idempotent e in a ring R is called right (left) semicentral if for every $x \in R$, $ex = exe$ ($xe = exe$). And the set of right (left) semicentral idempotents of R is denoted by $S_r(R)$ ($S_l(R)$). We define a ring R semiabelian if $Id(R) = S_r(R) \cup S_l(R)$.

All other necessary definitions and facts can be found in [1, 2, 3].

Theorem 1. *Let R be an semiabelian ring and A be an $n \times n$ idempotent matrix over R. If there exist elementary matrices $P_1, \ldots, P_k$ and $Q_1, \ldots, Q_s$ such that $P_1 \cdots P_k \cdot A \cdot Q_1 \cdots Q_s$ is a diagonal matrix, then there is elementary matrices $U_1, \ldots, U_l$ such that $U_1 \cdots U_l \cdot A \cdot (U_1 \cdots U_l)^{-1}$ is diagonal matrix.*

Theorem 2. *Let R be an semiabelian ring. Then a ring with elementary reduction of matrices is an EID-ring.*

Theorem 3. *The following are equivalent for a semialelian ring R:*

(a) *Each idempotent matrix over R is diagonalizable under a elementary transformation.*
(b) *Each idempotent matrix over R has a charateristic vector.*

Theorem 4. *Let R be an semiabelian ring, N be the set of nilpotents in R, and I be an ideal in R with $I \subseteq N$. Then R/I is an EID-ring, if and only if R is an EID-ring.*

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Higher power moments of the Riesz mean error term of hybrid symmetric square L-function

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Let $f(z) = \sum_{n=1}^{\infty} a_f(n) e^{2\pi i n z}$ be a holomorphic cusp form of even weight $k \geq 12$ for the full modular group $SL(2, \mathbb{Z})$, $z \in H$, $H = \{z \in \mathbb{C} | Im(z) > 0\}$ is the upper half plane. We suppose that $f(z)$ is a normalized eigenfunction for the Hecke operators $T(n) (n \geq 1)$ with $a_f(1) = 1$.

In [**1**], Shimura introduced the function $L(s, sym^2 f, \chi)$. For an arbitrary primitive Dirichlet character χ mod d, the hybrid symmetric square L-function attached to f is defined as the following Euler product:

$$L(s, sym^2 f, \chi) := \prod_p (1 - \alpha_f^2(p)\chi(p)p^{-s})(1 - \chi(p)p^{-s})^{-1} \times (1 - \overline{\alpha}_f^2(p)\chi(p)p^{-s})$$

for $\Re s > 1$.

Let $\Delta_\rho(t, sym^2 f, \chi)$ be the error term of the Riesz mean of the hybrid symmetric square L-function. We study the higher power moments of $\Delta_\rho(t, sym^2 f, \chi)$. Particularly, for $\rho = 1/2$, we prove the following result.

Theorem 1. *Let $X > 1$ be a real number. For any fixed $\epsilon > 0$, we have that*

$$\int_0^X \Delta_{\frac{1}{2}}^h(t, sym^2 f, \chi)dt = \frac{6B_{\frac{1}{2}}(h, c)d^{\frac{3h}{2}}}{(3+2h)(2\pi)^{\frac{3h}{2}}3^{\frac{h}{2}}} X^{\frac{2}{3}h+1} + O(X^{1+\frac{2}{3}h-\lambda_{\frac{1}{2}}(h,6)+\epsilon} d^{\frac{3h}{2}+\epsilon}),$$

holds for h = 3, 4, 5, where the O-constant depends on h and ϵ.

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Connection between automatic sequences and endomorphisms of rooted trees via d-adic dynamics

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The ring $\mathbb{Z}_d$ of d-adic integers has a natural interpretation as the boundary of a rooted d-ary tree T_d. Endomorphisms of this tree are in one-to-one correspondence with 1-Lipschitz mappings from $\mathbb{Z}_d$ to itself. Therefore, one can use the language of endomorphisms of rooted trees and, in particular, the language and techniques of Mealy automata (see, for example, [**5**]), to study such mappings. For example, polynomial transformations of $\mathbb{Z}_d$ in this context were studied in [**1**]. Each continuous transformation $f\colon \mathbb{Z}_d \to \mathbb{Z}_d$ can be decomposed into its van der Put series

$$f(x) = \sum_{n\geq 0} B_n^f \chi_n(x),$$

where $(B_n^f)_{n\geq 0} \subset \mathbb{Z}_d$ is a unique sequence of d-adic integers, and $\chi_n(x)$ is the characteristic function of the cylindrical set consisting of all d-adic integers with prefix $[n]_d$ (here by $[n]_d$ we mean the image of n in $\mathbb{Z}_d$ under the natural embedding $\mathbb{Z} \to \mathbb{Z}_d$ that is obtained by reversing the d-ary expansion of n). The coefficients B_n^f are called the *van der Put coefficients* of f and are computed as follows:

$$B_n^f = \begin{cases} f(n), & \text{if } 0 \leq n < d, \\ f(n) - f(n\_), & \text{if } n \geq d, \end{cases} \tag{1}$$