In [1], Shimura introduced the function $L(s, sym^2 f, \chi)$. For an arbitrary primitive Dirichlet character χ mod d, the hybrid symmetric square L-function attached to f is defined as the following Euler product:

$$L(s, sym^2 f, \chi) := \prod_p (1 - \alpha_f^2(p)\chi(p)p^{-s})(1 - \chi(p)p^{-s})^{-1} \times (1 - \overline{\alpha}_f^2(p)\chi(p)p^{-s})$$

for $\Re s > 1$.

Let $\Delta_\rho(t, sym^2 f, \chi)$ be the error term of the Riesz mean of the hybrid symmetric square L-function. We study the higher power moments of $\Delta_\rho(t, sym^2 f, \chi)$. Particularly, for $\rho = 1/2$, we prove the following result.

Theorem 1. *Let $X > 1$ be a real number. For any fixed $\epsilon > 0$, we have that*

$$\int_0^X \Delta_{\frac{1}{2}}^h(t, sym^2 f, \chi)dt = \frac{6B_{\frac{1}{2}}(h,c)d^{\frac{3h}{2}}}{(3+2h)(2\pi)^{\frac{3h}{2}}3^{\frac{h}{2}}} X^{\frac{2}{3}h+1} + O(X^{1+\frac{2}{3}h-\lambda_{\frac{1}{2}}(h,6)+\epsilon} d^{\frac{3h}{2}+\epsilon}),$$

holds for h = 3, 4, 5, where the O-constant depends on h and ϵ.

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Key words and phrases. Holomorphic cusp forms, symmetric square L-function

Connection between automatic sequences and endomorphisms of rooted trees via d-adic dynamics

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The ring $\mathbb{Z}_d$ of d-adic integers has a natural interpretation as the boundary of a rooted d-ary tree T_d. Endomorphisms of this tree are in one-to-one correspondence with 1-Lipschitz mappings from $\mathbb{Z}_d$ to itself. Therefore, one can use the language of endomorphisms of rooted trees and, in particular, the language and techniques of Mealy automata (see, for example, [5]), to study such mappings. For example, polynomial transformations of $\mathbb{Z}_d$ in this context were studied in [1]. Each continuous transformation $f\colon \mathbb{Z}_d \to \mathbb{Z}_d$ can be decomposed into its van der Put series

$$f(x) = \sum_{n\geq 0} B_n^f \chi_n(x),$$

where $(B_n^f)_{n\geq 0} \subset \mathbb{Z}_d$ is a unique sequence of d-adic integers, and $\chi_n(x)$ is the characteristic function of the cylindrical set consisting of all d-adic integers with prefix $[n]_d$ (here by $[n]_d$ we mean the image of n in $\mathbb{Z}_d$ under the natural embedding $\mathbb{Z} \to \mathbb{Z}_d$ that is obtained by reversing the d-ary expansion of n). The coefficients B_n^f are called the *van der Put coefficients* of f and are computed as follows:

$$B_n^f = \begin{cases} f(n), & \text{if } 0 \leq n < d, \\ f(n) - f(n\_), & \text{if } n \geq d, \end{cases} \tag{1}$$

where for $n = x_0 + x_1 d + \cdots + x_t d^t$ with $x_t \neq 0$ we define $n\_ = x_0 + x_1 \cdot d + \cdots + x_{t-1} d^{t-1} = n \bmod d^t$.

In the case when f is 1-Lipschitz, it was shown in [**4**] that $B_n^f = b_n^f d^{\lfloor \log_d n \rfloor}$ with $b_n^f \in \mathbb{Z}_d$. We will call the coefficients b_n^f the *reduced van der Put coefficients* of f.

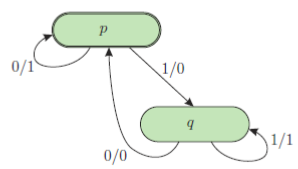
Anashin in [**3**] proved that a 1-Lipschitz transformation of $\mathbb{Z}_d$ defines a finite state endomorphism of T_d (i.e., can be defined by a finite Mealy automaton) if and only if the sequence of its reduced van der Put coefficients is made of eventually periodic d-adic integers and is d-automatic (i.e., can be defined by a finite Moore automaton, see Allouche's book [**2**] for details). We give an explicit constructive connection between the Moore automata accepting such a sequence and the Mealy automata inducing the corresponding endomorphism. This, in particular, gives a way to construct Mealy automata of mappings that are defined by automatic sequences, like Thue-Morse, for example.

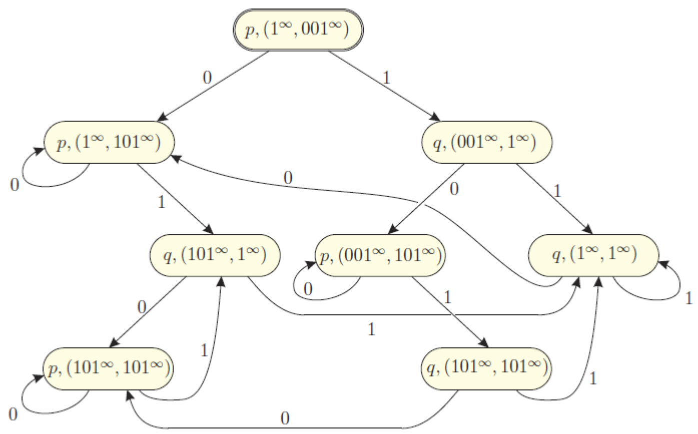
After explicitly describing the connection between Mealy and Moore automata we obtain the following results.

THEOREM 1. *Let g be an endomorphism of T_d defined by a finite Mealy automaton $\mathcal{A}$. Let also $(b_n^g)_{n\geq 0}$ be the (automatic) sequence of the reduced van der Put coefficients of a transformation $\mathbb{Z}_d \to \mathbb{Z}_d$ induced by g. Then the underlying oriented graph of the Moore automaton $\mathcal{B}$ defining $(b_n^g)_{n\geq 0}$ (possibly non-minimized) covers the underlying oriented graph of $\mathcal{A}$.*

THEOREM 2. *Let g be an endomorphism of T_d induced by a transformation of $\mathbb{Z}_d$ with the sequence of van der Put coefficients defined by finite Moore automaton $\mathcal{B}$. Then the underlying oriented graph of the Mealy automaton $\mathcal{A}$ defining g (possibly non-minimized) covers the underlying oriented graph of $\mathcal{B}$.*

For example, the figure below shows the Mealy automaton defining the lamplighter group $\mathcal{L} = \langle p, q \rangle$, and the corresponding Moore automaton defining the sequence of reduced van der Put coefficients of the d-adic transformation induced by its generator p.





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Key words and phrases. p-adic numbers, groups generated by automata, Mealy automata, Moore Automata, automatic sequences

This research was partially supported by the Simons Foundation through Collaboration Grant #317198.

Linear groups saturated by subgroups of finite central dimension

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Let F be a field, A be a vector space over F and G be a subgroup of $\mathrm{GL}(F,A)$. We say that G has a *dense family of subgroups, having finite central dimension*, if for every pair of subgroups H, K of G such that $H \leqslant K$ and H is not maximal in K there exists a subgroup L of finite central dimension such that $H \leqslant L \leqslant K$ (we can note that L can match with one of the subgroups H or K). We study the locally soluble linear groups with a dense family of subgroups, having finite central dimension.

Theorem 1. *Let F be a field, A be a vector space over F, having infinite dimension, and G be a locally soluble subgroup of* $\mathrm{GL}(F,A)$. *Suppose that G has infinite central dimension. If G has a dense family of subgroups, having finite central dimension, then G is a group of one of the following types:*

(i) *G is cyclic or quasicyclic p-group for some prime p;*
(ii) *$G = K \times L$ where K is cyclic or quasicyclic p-group for some prime p and L is a group of prime order;*
(iii) $G = \langle a, b \mid |a| = 2^n, |b| = 2, a^b = a^t$ *where* $t = 1 + 2^{n-1}, n \geqslant 3\rangle$;
(iv) $G = \langle a, b \mid |a| = 2^n, |b| = 2, a^b = a^t$ *where* $t = -1 + 2^{n-1}, n \geqslant 3\rangle$;
(v) $G = \langle a, b \mid |a| = 2^n, |b| = 2, a^b = a^{-1}\rangle$;
(vi) $G = \langle a, b \mid |a| = 2^n, b^2 = a^t$ *where* $t = 2^{n-1}, a^b = a^{-1}\rangle$;
(vii) $G = \langle a, b \mid |a| = p^n, |b| = p, a^b = a^t, t = 1 + p^{n-1}, n \geqslant 2\rangle$, *$p$ is an odd prime;*
(viii) $G = \langle a\rangle \leftthreetimes \langle b\rangle$, $|a| = p^n$ *where p is an odd prime,* $|b| = q$, *q is a prime,* $q \neq p$;
(ix) $G = B \leftthreetimes \langle a\rangle$, $|a| = p^n$, $B = C_G(B)$ *is an elementary abelian q-subgroup, p and q are primes,* $p \neq q$, *B is a minimal normal subgroup of G;*
(x) $G = K \leftthreetimes \langle b\rangle$, *where K is a quasicyclic 2-subgroup,* $|b| = 2$ *and* $x^b = x^{-1}$ *for each element* $x \in K$;
(xi) $G = K\langle b\rangle$, *where* $K = \langle a_n \mid a_1^p = 1, a_{n+1}^p = a_n, n \in \mathbb{N}\rangle$ *is a quasicyclic 2-subgroup,* $b^2 = a_1$ *and* $a_n^b = a_n^{-1}$, $n \geqslant 2$;
(xii) $G = K \leftthreetimes \langle b\rangle$, *where K is a quasicyclic p-subgroup, p is an odd prime,* $K = C_G(K)$, $|b| = q$ *is a prime such that* $p \neq q$;