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# Linear groups saturated by subgroups of finite central dimension

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Let $F$ be a field, $A$ be a vector space over $F$ and $G$ be a subgroup of $\mathrm{GL}(F, A)$. We say that $G$ has a *dense family of subgroups, having finite central dimension*, if for every pair of subgroups $H$, $K$ of $G$ such that $H \leqslant K$ and $H$ is not maximal in $K$ there exists a subgroup $L$ of finite central dimension such that $H \leqslant L \leqslant K$ (we can note that $L$ can match with one of the subgroups $H$ or $K$). We study the locally soluble linear groups with a dense family of subgroups, having finite central dimension.

Theorem 1. *Let $F$ be a field, $A$ be a vector space over $F$, having infinite dimension, and $G$ be a locally soluble subgroup of $\mathrm{GL}(F, A)$. Suppose that $G$ has infinite central dimension. If $G$ has a dense family of subgroups, having finite central dimension, then $G$ is a group of one of the following types:*

(i) *$G$ is cyclic or quasicyclic $p$-group for some prime $p$;*
(ii) *$G = K \times L$ where $K$ is cyclic or quasicyclic $p$-group for some prime $p$ and $L$ is a group of prime order;*
(iii) $G = \langle a, b \mid |a| = 2^n, |b| = 2, a^b = a^t$ *where* $t = 1 + 2^{n-1}, n \geqslant 3\rangle$;
(iv) $G = \langle a, b \mid |a| = 2^n, |b| = 2, a^b = a^t$ *where* $t = -1 + 2^{n-1}, n \geqslant 3\rangle$;
(v) $G = \langle a, b \mid |a| = 2^n, |b| = 2, a^b = a^{-1}\rangle$;
(vi) $G = \langle a, b \mid |a| = 2^n, b^2 = a^t$ *where* $t = 2^{n-1}, a^b = a^{-1}\rangle$;
(vii) $G = \langle a, b \mid |a| = p^n, |b| = p, a^b = a^t, t = 1 + p^{n-1}, n \geqslant 2\rangle$, *$p$ is an odd prime;*
(viii) $G = \langle a\rangle \leftthreetimes \langle b\rangle$, $|a| = p^n$ *where $p$ is an odd prime,* $|b| = q$, *$q$ is a prime,* $q \neq p$;
(ix) $G = B \leftthreetimes \langle a\rangle$, $|a| = p^n$, $B = C_G(B)$ *is an elementary abelian $q$-subgroup, $p$ and $q$ are primes, $p \neq q$, $B$ is a minimal normal subgroup of $G$;*
(x) $G = K \leftthreetimes \langle b\rangle$, *where $K$ is a quasicyclic 2-subgroup,* $|b| = 2$ *and* $x^b = x^{-1}$ *for each element* $x \in K$;
(xi) $G = K\langle b\rangle$, *where* $K = \langle a_n \mid a_1^p = 1, a_{n+1}^p = a_n, n \in \mathbb{N}\rangle$ *is a quasicyclic 2-subgroup,* $b^2 = a_1$ *and* $a_n^b = a_n^{-1}$, $n \geqslant 2$;
(xii) $G = K \leftthreetimes \langle b\rangle$, *where $K$ is a quasicyclic $p$-subgroup, $p$ is an odd prime,* $K = C_G(K)$, $|b| = q$ *is a prime such that* $p \neq q$;

(xiii) $G = Q \leftthreetimes K$, *where $K$ is a quasicyclic $p$-subgroup, $Q = C_G(Q)$ is an elementary abelian $q$-subgroup, $p$, $q$ are primes, $p \neq q$, $Q$ is a minimal normal subgroup of $G$.*

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# Semiscalar equivalence of one class of 3-by-3 matrices

Bohdan Shavarovskii

Let a matrix $F(x) \in M(3,\ \mathbb{C}[x])$ have a unit first invariant factor and only one characteristic root. We assume that this uniquely characteristic root is zero. In [1], the author proved that in the class $\{PF(x)Q(x)\}$, where $P \in GL(3,\ C)$, $Q(x) \in GL(3,\ \mathbb{C}[x])$ there exists a matrix

$$A(x) = \left\| \begin{array}{ccc} 1 & 0 & 0 \\ a_1(x) & x^{k_1} & 0 \\ a_3(x) & a_2(x) & x^{k_2} \end{array} \right\|$$

(notation: $A(x) \approx F(x)$), which has the following properties:

$(i)$ $\deg a_1 < k_1,\ \deg a_2,\ \deg a_3 < k_2,\ a_2(x) = x^{k_1} a_2'(x),\ a_1(0) = a_2'(0) = a_3(0) = 0;$

$(ii)$ $co\deg a_3 \neq co\deg a_1,\ co\deg a_2'$, if $co\deg a_3 < co\deg a_2$;

$(iii)$ $co\deg a_3 \neq 2co\deg a_1 + co\deg a_2'$ and in $a_1(x)$ the monomial of the degree $2co\deg a_1$ is absent, if $co\deg a_3 \geq co\deg a_2$.

Here $co\deg$ denotes *the junior degree* of polynomial. The purpose of this report is to construct the canonical form of the matrix $F(x)$ in the class $\{PF(x)Q(x)\}$. If both elements $a_1(x),\ a_2(x)$ of the matrix $A(x)$ are non-zero, then we may take their junior coefficients to be identity elements. In the opposite case, we may take the junior coefficients of the non-zero subdiagonal elements of the matrix $A(x)$ to be one. Such matrix $A(x)$ in [1] is called the *reduced matrix*. In this report we consider the case, when some of the elements $a_1(x),\ a_2(x),\ a_3(x)$ of the matrix $A(x)$ are equal to zero and at least one of them is different from zero.

Theorem 1. *Let in the reduced matrix $A(x)$ the conditions $a_i(x) \neq 0$, for some index $i$ from set $\{1,\ 2,\ 3\}$ and $a_j(x) \equiv 0$ for the rest $j \in \{1,\ 2,\ 3\}$, $j \neq i$, be fulfilled. Then $A(x) \approx B(x)$, where in the reduced matrix*

$$B(x) = \left\| \begin{array}{ccc} 1 & 0 & 0 \\ b_1(x) & x^{k_1} & 0 \\ b_3(x) & b_2(x) & x^{k_2} \end{array} \right\|,$$